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APPLICATION DE LA MÉTHODE INTERFÉRENTIELLE À LA MESURE DE TRÈS PETITS DÉPLACEMENTS DE RAIES. COMPARAISON DU SPECTRE SOLAIRE AVEC LE SPECTRE D'ARC DU FER. COMPARAISON DU CENTRE ET DU BORD DU SOLEIL

PAR CH. FABRY ET H. BUISSON

Un grand nombre de problèmes d'astrophysique conduisent à étudier de petits changements ou de petites différences de longueurs d'onde. La méthode ordinairement employée consiste à faire des mesures micrométriques de positions de raies dans un spectre très dispersé. Cela ne va pas toujours sans difficultés; un certain nombre de causes d'erreur sont à éviter: déplacements ou déformations de l'appareil, dissymétries d'éclairement, variations de température. Dans certains cas, on a été amené à employer un artifice délicat pour surmonter ces difficultés: comparaison de la raie variable avec des raies telluriques très voisines; cela n'est évidemment applicable que dans des cas très particuliers et l'on peut avoir un léger doute sur la fixité des repères lorsque la hauteur du soleil varie.

Les méthodes interférentielles permettent de mesurer de petites variations de la longueur d'onde d'une radiation en valeur absolue, sans faire intervenir aucun repère. La chose invariable est une dimension matérielle (épaisseur d'un étalon) dont il est facile de maintenir et de contrôler la constance. La précision et l'invariabilité de l'appareil dispersif n'interviennent pas: cet appareil ne sert qu'à séparer les radiations.

Nous avons fait quelques applications de ces méthodes. Nous nous proposons, dans ce mémoire, d'en faire la théorie, de décrire les appareils employés, et d'exposer les résultats obtenus.

I. MÉTHODE ET APPAREILS

Le dispositif employé se compose de deux parties: l'appareil interférentiel, et l'appareil dispersif qui sépare les interférences dues aux diverses radiations.

L'appareil interférentiel est un système de deux surfaces argentées maintenues au parallélisme et à distance invariable pendant les expériences. Le plus souvent c'était un *étalon* de la forme construite par Jobin.[1] Il fallait le mettre à l'abri des variations brusques de température, en particulier de celles qui peuvent être produites par la présence des observateurs; il a suffi pour cela de le placer dans une boite en carton munie des ouvertures convenables pour laisser passer la lumière.[2] Nous avons employé des épaisseurs de 2. 5, 5, et 10 mm. Exceptionnellement, pour d'autres épaisseurs, nous avons employé l'interféromètre.[3]

Chaque radiation monochromatique qui traverse la lame d'air donne un système d'anneaux à l'infini; un objectif de 26 cm de foyer en projette l'image dans son plan focal.

Un spectroscope sans astigmatisme placé ensuite sépare les aspects relatifs à chaque radiation. Quand le spectre est à raies très nombreuses, la dispersion de ce spectroscope doit être grande pour bien séparer les diverses raies. Nous avons presque toujours employé un spectroscope du type autocollimateur à réseau plan de Rowland (568 traits par mm, 8×5 cm de partie striée), avec objectif de 3.10 m de foyer. On fait défiler le spectre par simple rotation du réseau, au moyen de cordons que l'observateur a sous la main. Ce type de spectroscope est très commode et très peu encombrant: il tient tout entier sur une poutre de 3.50 m de long et 25 cm de large. Un inconvénient provient de la lumière réfléchie par l'objectif, qui donne un fond lumineux dans le champ. Avec l'objectif achromatique que

[1] *Astrophysical Journal*, **15**, 81, 1902.

[2] Les étalons dont nous nous sommes servis étaient en acier. L'emploi d'étalons en invar aurait eu de grands avantages.

[3] *Astrophysical Journal*, **13**, 265, 1901.

nous employons, cette lumière réfléchie se partage en trois faisceaux dont l'un donne une image réelle de la fente et les deux autres des images virtuelles. L'image réelle, qui se trouve à 60 cm en avant de l'objectif, est interceptée par un fil métallique de 1 mm de diamètre. La lumière de l'une des images virtuelles, située prés de l'objectif et en arrière, est arrêtée par un petit écran de 15×6 mm collé sur l'objectif. Enfin, en inclinant très légérement l'objectif, on rejette en dehors du champ le troisième faisceau, qui est très peu divergent. On arrive ainsi à s'affranchir de la lumière réfléchie, sans diminuer sensiblement la surface utilisée du réseau.

Nous utilisons ordinairement le troisième spectre, ce qui donne une dispersion de 0.58 mm par ångström.

Dans certains cas, pour avoir plus de lumière, nous avons employé un spectroscope autocollimateur à prismes, avec objectif de 1 mètre de distance focale, et deux prismes de flint.[1] Dans la région 4300 un ångström occupe 0.15 mm.

Dans tous les cas, la fente du spectroscope est dans le plan focal de l'objectif qui projette les anneaux, suivant un diamètre de ceux-ci. L'arrangement est celui qui nous a servi pour la mesure des longueurs d'onde des repères fondamentaux du spectre.[2]

La surface utilisée de l'appareil interférentiel est très petite: elle est limitée par un écran percé d'une ouverture, dont les dimensions n'excèdent pas quelques millimètres. La distance de cette ouverture à l'objectif qui projette les anneaux est telle qu'une image réelle en soit projetée, à travers la fente, sur le réseau. De cette manière, toute la lumière qui a traversé l'ouverture et la fente est finalement utilisée par le spectroscope.

Théorie des interférences produites par les raies noires d'un spectre.— Dans le cas d'un spectre à raies brillantes, la théorie du phénomène ne présente aucune difficulté: chaque image monochromatique de la fente est réduite à un certain nombre de points brillants, intersections de la fente avec les anneaux d'interférence produits par la radiation correspondante. La largeur maxima de la fente n'est limitée que par la condition que les images des différentes raies n'empiètent pas.

[1] *Journal de Physique* (4), **3**, 204, 1904.

[2] *Astrophysical Journal*, **28**, 169, 1908.

Dans le cas du spectre solaire, on a un spectre continu avec des raies noires, et les mesures doivent porter sur celles-ci. On peut alors, dans des conditions convenables, obtenir l'aspect complémentaire de celui qu'on a avec des raies brillantes, comme si l'on avait des interférences produites par les raies noires, c'est-à-dire par des radiations absentes dans le spectre. Ce phénomène, en apparence paradoxal, s'explique par les considérations suivantes.

A travers l'appareil disposé comme il a été indiqué ci-dessus, faisons passer une lumière donnant un spectre rigoureusement continu. Supposons le spectroscope à fente infiniment fine, et ayant un pouvoir de définition infini, de telle manière qu'à chaque point du champ corresponde une radiation rigoureusement définie. Dans le spectre, la lumière se répartit alors en lignes brillantes formant des cannelures légèrement courbes: d'un point à un autre de la fente, la différence de marche varie, elle est maximum au point où se projette le centre des anneaux, et décroît de part et d'autre comme le carré de la distance à ce point. A mesure que l'épaisseur de l'appareil interférentiel augumente, les franges deviennent plus serrées. Ces cannelures ont l'aspect ordinaire aux franges des lames argentées: ce sont des lignes brillantes dont la largeur est faible par rapport à celle des espaces noirs qui les séparent; cet effet est d'autant plus marqué que le pouvoir réflecteur est plus élevé. En somme, pour chaque radiation, c'est-à-dire pour chaque ligne verticale du spectre, les interférences ramassent la lumière en certains points.

Si l'on élargit la fente, les lignes brillantes qui forment les cannelures s'élargissent d'une quantité égale à la largeur de la fente, et lorsque chaque bande brillante rejoint la bande voisine les cannelures disparaissent. L'aspect est devenu celui d'un simple spectre continu, comme si l'appareil interférentiel était enlevé; toutefois, la constitution de ce spectre est très différente de celle d'un spectre continu avec fente large. Dans ce dernier cas, chaque radiation monochromatique se répartit uniformément sur un rectangle, image de la fente, et en chaque point on a un mélange de radiations. Au contraire, dans le spectre dont l'aspect est devenu continu par disparition des cannelures, chaque radiation occupe seulement des lignes horizontales dont la longueur est égale à la largeur de la fente. Les radiations voisines s'échelonnent en hauteur, avec un très petit décalage horizontal

correspondant à la dispersion du spectroscope. Les petits traits de lumière monochromatique remplissent tout le champ lorsque la fente a la largeur voulue, et l'œil, qui n'est pas un appareil spectroscopique, ne distingue pas le spectre ainsi obtenu d'un spectre continu ordinaire.

Pour une largeur de la fente plus grande que celle-là, les cannelures reparaissent, chaque trait empiétant sur le suivant.

D'autre part, pour un pouvoir de définition limité, les cannelures ont sensiblement leur aspect théorique lorsqu'elles sont peu serrées (différence de marche faible); si la différence de marche va en augmentant, les cannelures perdent d'abord leur aspect de bandes brillantes fines, s'estompent, et finissent par disparaître lorsque leur intervalle tombe au dessous du pouvoir de définition du spectroscope. On supposera que l'on n'arrive pas à ce cas.

Supposons maintenant une raie noire dans le spectre. Elle est forcément de largeur finie, c'est-à-dire que toutes les radiations comprises entre deux limites déterminées sont absentes, ou d'intensité négligeable. Avec une fente étroite, on aura les cannelures, lignes brillantes fines, coupées par la raie noire. Celle-ci ne se manifeste qu'aux points où elle rencontre une cannelure brillante, par une interruption de la cannelure. Cet aspect n'est pas commode pour les mesures, et d'ailleurs la luminosité est faible. Elargissons la fente. Cela revient à juxtaposer des aspects analogues au précédent, mais déplacés dans le sens horizontal. Les cannelures élargies sont alors coupées par des traits noirs horizontaux. On a alors dans le spectre deux espèces d'intervalles sombres: les intervalles entre les cannelures brillantes, et les traits noirs relatifs aux radiations absentes; ce sont ces derniers qui sont intéressants pour les mesures; il y a intérêt à faire disparaître les premiers. C'est ce qui a lieu lorsque la fente a la largeur qui fait disparaître les cannelures. Il ne reste plus alors que les traits noirs, dont la largeur dans le sens horizontal est égale à la largeur de la fente, alignés verticalement sur l'image de la raie, et qui se détachent sur un fond uniforme.

Ce que l'on a dit plus haut sur la constitution du spectre continu dont on a fait disparaître les cannelures fait d'ailleurs voir immédiatement ce dernier résultat: les diverses radiations sont séparées en hauteur, et l'on obtient des traits noirs correspondant à celles qui manquent.

L'épaisseur, dans le sens vertical, des traits noirs correspondant à une raie, dépend de la largeur de celle-ci. On obtient ainsi des rectangles noirs; lorsque l'épaisseur de chacun est devenue assez grande pour qu'il rejoigne le rectangle voisin, les interférences cessent d'être visibles, et la raie est uniformément noire dans le sens de sa hauteur. Cela arrive lorsque l'ordre d'interférence varie d'une unité dans la largeur de la raie noire. La largeur de la raie, dans un spectre parfaitement pur, est alors égale à la distance de deux cannelures. Avec un appareil interférential d'épaisseur e, la largeur maxima $d\lambda$ d'une raie pouvant donner des interférences est donnée par

$$\frac{d\lambda}{\lambda}=\frac{\lambda}{2e}=\frac{1}{p}$$

p étant l'ordre d'interférence. Inversement, pour une raie donnée, les interférences cesseront d'être observables lorsque l'épaisseur de l'appareil interférentiel dépassera la limite e donnée par la même équation.

Dans tout ce qui précède, on a raisonné comme si le pouvoir réflecteur des lames argentées était égal à 1, ce qui produit des cannelures infiniment nettes. En réalité il n'en est pas ainsi; des cannelures sont légèrement estompées. Il en résulte une limite du pouvoir de définition interférentiel, tout-à-fait analogue à la limite du pouvoir de résolution spectroscopique, et qui fait que les raies de largeur inférieure à une certaine limite ne sont pas visibles pour une différence de marche donnée. Il en résulte que, pour l'observation d'une raie donnée, l'épaisseur de l'appareil interférentiel ne doit pas descendre au dessous d'une certaine limite, qui peut être évaluée à un dixième de la limite supérieure.

Calcul et observations.—Supposons que l'on ait à étudier de petits déplacements d'une raie; soient λ et λ' les deux valeurs successives de la longueur d'onde. Si l'on mesure les diamètres d'un même anneau correspondant à ces deux valeurs, on peut calculer la variation de longueur d'onde $\lambda'-\lambda$. Soit en effet l'anneau d'ordre P, a et a' ses diamètres angulaires successifs. L'ordre d'interférence au centre est, dans le premier cas,

$$p=P\left(1+\frac{a^2}{8}\right)=\frac{2e}{\lambda},$$

en appelant e la distance des deux surfaces argentées.

De même, avec la radiation λ'

$$p' = P\left(1 + \frac{a'^2}{8}\right) = \frac{2e}{\lambda'}.$$

On en déduit facilement:

$$\lambda' - \lambda = \lambda \frac{a^2 - a'^2}{8}.$$

a et a' sont les diamètres angulaires exprimés en radians. Si l'on mesure les diamètres linéaires d et d' d'images réelles des anneaux projetées au moyen d'un objectif de distance focale f, on aura:

$$\lambda' - \lambda = \lambda \frac{d^2 - d'^2}{8f^2}.$$

La seule détermination qui doive être faite avec précision est celle de la différence des diamètres d'anneaux.

Aucune des corrections qui interviennent dans les mesures absolues (changement de phase par réflexion, dispersion de l'air) n'est à considérer.

Les observations peuvent être faites visuellement ou par photographie.

Dans les observations visuelles, on mesure directement les diamètres angulaires: l'appareil interférentiel peut subir de petites rotations autour d'un axe horizontal perpendiculaire au faisceau lumineux, ce qui déplace le centre des anneaux sur la fente. On peut ainsi amener successivement les deux extrémités du diamètre d'un anneau sur un fil horizontal fixe placé dans le plan du spectre. L'angle dont il faut faire tourner l'appareil interférentiel est égal au diamètre angulaire de l'anneau. On le mesure en visant, avec une lunette fixe, dans un miroir lié à l'appareil interférentiel, l'image d'une échelle divisée.

Dans le cas d'observations photographiques, on mesure sur le cliché les diamètres linéaires des anneaux au moyen d'un comparateur.

Dans la comparaison du spectre solaire avec celui d'une source terrestre, il faut tenir compte du déplacement de l'observateur par rapport au soleil.

II. COMPARAISON DES LONGUEURS D'ONDE DES SPECTRES DE L'ARC ET DU SOLEIL

La figure 1 représente le dispositif employé pour l'utilisation du faisceau solaire.

A est un héliostat polaire, muni d'un miroir plan de 20 cm de diamètre. Le faisceau, renvoyé suivant l'axe du monde, tombe sur l'objectif *B* de 3 m de foyer, et traverse un tunnel percé dans la muraille de la salle. Il se réfléchit sur le miroir *C* qui le renvoie verticalement, et enfin sur le prisme à réflexion totale *D* qui le dirige horizontalement; en *F* on obtient l'image réelle du soleil, qui a 28 mm de diamètre. Ce dispositif est très commode dans les conditions où nous nous trouvons: il tient très peu de place; en projection horizontale, il n'y a qu'une très petite distance entre l'héliostat et l'image du soleil. Les miroirs *A* et *C* sont de bonne qualité; le second, qui reçoit un faisceau déja notablement rétréci n'a que 8 cm de diamètre. La troisième réflexion se fait sur un prisme à réflexion totale; celui-ci ne gâte pas l'image, qui se forme très près de lui. Cette dernière réflexion permet de renvoyer la lumière dans n'importe quelle direction du plan horizontal. L'image du soleil peut subir de petits déplacements dans son plan par de petites variations d'orientation du miroir *C*.

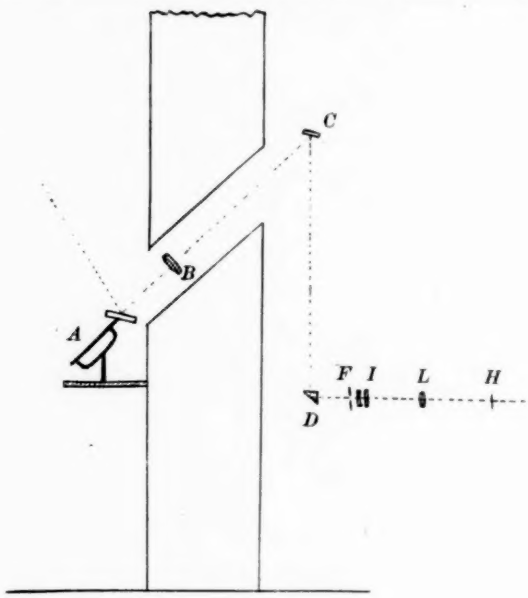

FIG. 1

Dans le plan *F*, où vient se former l'image du soleil, est un écran percé d'une ouverture qui limite la région utilisée du soleil. Immédiatement après est placé l'appareil interférentiel *I*. L'image des anneaux est projetée sur la fente *H* du spectroscope au moyen de la lentille *L*.

Echauffement de l'appareil interférentiel dû au faisceau solaire.— Nous avons rencontré une difficulté due à l'échauffement, par le

faisceau solaire, des lames de verre de l'appareil interférentiel: dès que ce faisceau traverse l'appareil, les surfaces se déforment. La lame d'air n'est plus limitée par des surfaces planes, et son épaisseur dans la partie centrale subit une diminution notable. Nous sommes arrivés à nous affranchir complètement de cette difficulté par l'emploi d'un certain nombre de précautions:

1. Des lames de quartz se déforment infiniment moins que des lames de verre, parce qu'étant beaucoup moins absorbantes pour l'infra-rouge, elles ne s'échauffent presque pas.

2. On peut diminuer beaucoup l'échauffement des lames de verre en absorbant l'infra-rouge. La substance qui nous a donné le meilleur résultat est une solution de sulfate de cuivre à 4 pour 100 sous une épaisseur de 16 mm. L'énergie totale de la radiation solaire est réduite au sixième de sa valeur, et dans le spectre visible l'absorption n'est sensible que pour les parties extrêmes du spectre.

3. Il est rationnel de ne laisser passer à travers l'appareil interférentiel que la lumière réellement utilisée par la suite. Or, sur la fente du spectroscope se forme une image presque nette de l'objectif *B* (fig. 1) placé à 3 m de la lentille de court foyer qui projette les anneaux sur la fente. On peut donc, sans inconvénient, diaphragmer beaucoup l'objectif *B* dans le sens de la largeur. Nous laissons libre seulement une ouverture rectangulaire de 1 cm de large sur 12 de long. Dans le plan de la fente du spectroscope, il reste une bande lumineuse étroite à bords un peu flous, dont la fente occupe le milieu.

Arc.—On s'est borné à étudier le spectre du fer. Il est produit dans les mêmes conditions que celles employées pour obtenir notre atlas de ce spectre:[1] arc entre tiges de fer de 7 mm de diamètre, alimenté en courant continu sous tension de 220 volts, avec des intensités de courant variables selon les cas.

Dans la suite de cette étude (voir plus loin) nous avons été amenés à employer l'arc dans le vide. Nos premières expériences ont été faites avec un appareil improvisé de la manière suivante: deux tiges de fer verticales sont placées dans un ballon de verre; l'une est mastiquée dans une tubulure à la partie inférieure; l'autre peut coulisser dans une seconde tubulure placée à la partie supérieure, pour l'al-

[1] *Annales de la Faculté des sciences de Marseille*, **17**, 111. Hermann, éditeur, Paris 1909.

lumage et le réglage de l'arc. Une troisième ouverture permet de faire le vide; un col horizontal, fermé par une glace laisse passer la lumière. On évite l'échauffement par une circulation d'eau sur les parois du ballon, ou en l'immergeant complètement dans l'eau.

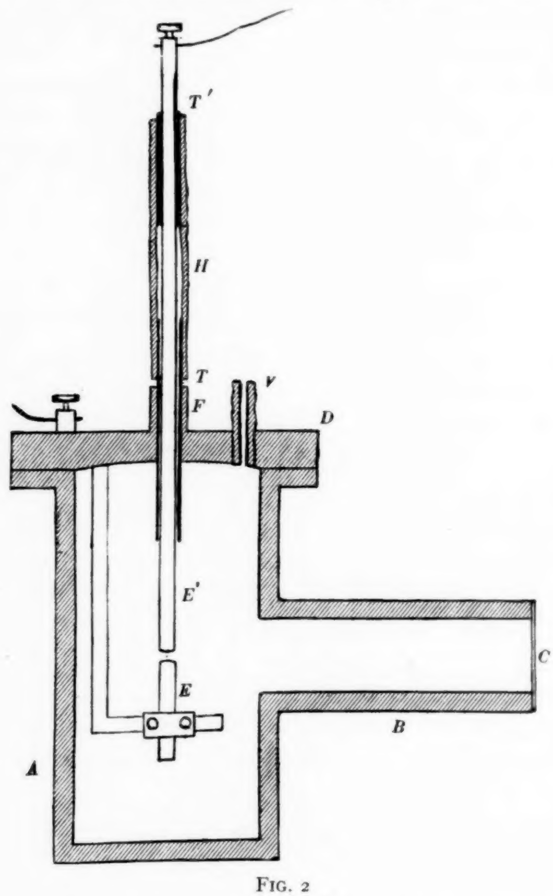

FIG. 2

Nous avons depuis construit un appareil plus commode (fig. 2). Le cylindre de fonte *A* porte un col horizontal *B* fermé par une lame de verre ou de quartz *C* qui laisse passer la lumière. Le couvercle rodé *D* porte une ouverture *V* reliée à la pompe. L'une des électrodes *E* est fixée au couvercle par l'intermédiaire d'une pince qui permet de la mettre en place. L'autre électrode *E'* peut glisser verticalement:

elle est mastiquée dans le tube de verre T' et passe librement à travers le tube de verre T mastiqué dans la tubulure F du couvercle. Les deux tubes de verre TT' sont reliés par le tube de caoutchouc H qui assure l'étanchéité, et laisse un jeu suffisant pour l'allumage et le réglage de l'arc.

A basse pression, l'arc est stable si les électrodes sont couvertes d'une goutte d'oxyde fondu, qui se forme spontanément lorsque l'arc est produit à l'air libre. Nous avons opéré sous une pression de quelques millimètres. Dans ces conditions, l'arc est beaucoup moins lumineux qu'à la pression atmosphérique. Nous employons un courant de 8 ampères.

Mode opératoire.—L'arc est placé latéralement par rapport au faisceau solaire. Son image est projetée sur la petite ouverture qui est devant l'appareil interférentiel, au moyen d'une lentille et par réflexion sur un prisme à réflexion totale. L'appareil reçoit la lumière de l'arc ou celle du soleil selon que le prisme est mis en place ou enlevé.

L'ouverture du diaphragme qui limite le faisceau est un rectangle de 8 mm sur 5; elle est telle que son image couvre complètement le réseau. L'image du soleil est centrée sur l'ouverture, de façon à éliminer toute influence de la rotation solaire.

Dans le cas de mesures visuelles, on fait une série de pointés alternativement avec les deux sources. Pour les mesures photographiques, on fait deux poses avec une des sources, séparées par une pose avec l'autre. Si l'appareil interférentiel n'a subi aucune variation, la première et la dernière pose doivent donner des diamètres d'anneaux identiques. En réalité, la différence entre les deux valeurs de l'ordre d'interférence au centre ne dépasse pas 0.02. On prend la moyenne des résultats des mesures obtenues sur les deux poses extrêmes, pour la comparer avec le résultat de la mesure sur la pose intermédiaire.

Comparaison du spectre solaire avec celui de l'arc dans l'air.—Il a été soupçonné depuis longtemps qu'il existe des différences entre les longueurs d'onde dans l'arc et le soleil. Ces différences se sont montrées dans le travail de Rowland. Une série de mesures comparatives a été faite par Jewell.[1] D'une façon générale, les longueurs d'onde dans le spectre solaire se trouvent un peu plus grandes que dans

[1] *Astrophysical Journal*, **3**, 89, 1896.

le spectre de l'arc, mais avec de nombreuses exceptions. L'accroissement de longueur d'onde lorsqu'on passe de l'arc au soleil, peut être expliqué par la pression de la couche renversante, et la valeur du déplacement peut donner la valeur de cette pression. Mais l'existence de déplacements de sens inverse laisse un doute sur l'explication et sur les résultats.

Dans nos mesures, nous ne nous sommes pas attachés à étudier le plus grand nombre de raies possible. D'autre part, nous avons éliminé toutes les fortes raies, qui sont trop larges pour des mesures précises.

Nous avons étudié par photographie la région 4530–4060; c'est à cette région que se rapporte le travail de Duffield[1] sur le déplacement des raies par la pression. Visuellement, nous avons étudié la région verte, et en outre quelques raies dans le jaune et le rouge.

Les nombres du tableau suivant sont des moyennes de résultats obtenus, le plus souvent, avec des épaisseurs différentes d'appareil interférentiel: nous avons employé des épaisseurs de 2.5 et 5 mm.

Pour un certain nombre de raies, ce tableau renferme (3^{me} colonne) une valeur de la différence soleil – arc *déduite des mesures absolues*. Les nombres de cette colonne ont été obtenus en comparant les valeurs absolues mesurées dans les deux spectres au moyen d'expériences complètement indépendantes: (1) dans le spectre solaire, valeurs absolues obtenues en 1900 par Perot et Fabry;[2] (2) dans le spectre de l'arc, valeurs absolues déterminées par Fabry et Buisson[3] en 1908, dont la plupart ont été publiées, et quelques-unes mesurées plus tard sur les mêmes clichés. Il y a une concordance très satisfaisante entre les différences directement mesurées et celles obtenues de cette manière indirecte. Cela constitue une vérification intéressante de l'exactitude des diverses séries de mesures.

Quelques-unes de nos raies ont été aussi étudiées par Jewell: la quatrième colonne donne ses résultats, qui sont en accord satisfaisant avec les nôtres.

Dans ce tableau, comme dans tous les suivants, les différences de longueurs d'onde sont exprimées en millièmes d'ångström.

[1] *Philosophical Transactions*, A, **208**, 111, 1908.

[2] *Astrophysical Journal*, **15**, 73, 261, 1902.

[3] *Ibid.* **28**, 169, 1908.

TABLEAU I

Longueurs d'onde (système internat.)	$\lambda_\odot - \lambda_{arc}$	Mesures absolues	Jewell	Groupe*	Longueurs d'onde (système internat.)	$\lambda_\odot - \lambda_{arc}$	Mesures absolues	Jewell	Groupe*
4062.45	+ 4		±0	S†	4859.75	− 7			DR
4118.55	+ 8			S	4871.34	−15			DR
4127.61	+ 2			S	4919.00	− 8			DR
4134.68	+ 5			S	5123.73	+10	+12		S
4153.92	−25			DR‡	5171.61	+25	+21		
4154.51	+ 3			S	5266.57	± 0			DR
4154.82	− 6			DR	5269.55	+12			S
4158.80	+ 8			S	5281.80	− 8			DR
4175.65	+ 4			S	5283.63	+ 5			S
4181.76	+ 2			S	5302.31	− 6			DR
4187.04	− 3			DR	5324.19	± 0			DR
4191.44	− 3			DR	5339.95	−11			DR
4202.04	+18		+ 6		5341.03	+18			S
4216.19	+11			S	5364.88	+30			DV
4222.22	−10		−14	DR	5365.41	± 0			DR
4227.44	−29			DR	5367.48	+30	+27		DV
4233.61	− 8			DR	5369.98	+22			DV
4235.95	−19			DR	5383.38	+23			DV
4250.12	−14			DR	5393.18	− 8			DR
4250.78	+23			DV§	5397.15	+13			S
4271.16	−18			DR	5405.78	+15			S
4282.41	+ 7			S	5410.92	+26			DV
4337.05	+ 7			S	5415.22	+27			DV
4352.74	+ 4		+ 7	S	5424.09	+30			DV
4369.77	+ 6		+ 4	S	5434.53	+ 7	+12		S
4375.93	+12		+10	S	5497.52	+ 5	+13		S
4422.57	+ 4			S	5501.47	+ 9			S
4427.31	+ 4			S	5506.78	+ 8	+ 9		S
4430.61	+ 6			S	5586.77	+ 3	+ 6		S
4442.34	+12			S	5763.01	−14	−11		DR
4443.19	+ 7			S	5934.68	−25	−19		DR
4447.72	+ 7		+ 6	S	5987.08	+19	+15		
4461.65	+ 9			S	6065.49	+10	+11		S
4466.55	+14			S	6230.73	+12	+11		S
4531.15	+ 5			S	6393.61	+ 4			S
4786.83	+ 6			S	6408.03	−20	−22		DR
4789.66	+ 8			S					

* La 5me colonne donne l'indication des groupes auxquels appartiennent les diverses raies (voir plus loin).

† S désigne les raies à élargissement symétrique.

‡ DR désigne les raies qui s'élargissent vers le rouge.

§ DV désigne les raies qui s'élargissent vers le violet.

En résumé, l'existence des différences entre le soleil et l'arc est incontestable. Les déplacements sont très différents d'une raie à une autre. Dans la majorité des cas, quand on passe de l'arc au soleil, on a un accroissement de longueur d'onde de quelques millièmes d'ångström, mais d'assez nombreuses raies se comportent autre-

ment: les unes donnent un déplacement de même sens, mais beaucoup plus grand, s'élevant jusqu'à 0.030 ångström; les autres donnent un déplacement très notable en sens inverse. Le déplacement par la pression ne suffit évidemment pas à expliquer ces résultats. D'autres causes agissent certainement, et si l'on veut pouvoir calculer la pression de la couche renversante il faut d'abord déterminer quelles sont les autres causes de déplacement. Nous les avons trouvées dans l'étude de l'élargissement des raies d'émission.

Elargissement des raies.—Aucune raie de l'arc n'est infiniment fine, et la largeur dépend de diverses circonstances. En particulier, les raies s'élargissent quand on augmente l'intensité du courant ou la pression de l'atmosphère ambiante. Les raies les plus fines sont obtenues en produisant l'arc dans le vide. L'observation des limites d'interférences donne une mesure de la largeur des raies. Dans le vide, toutes les raies de l'arc au fer ont à peu près la même largeur, d'environ 0.03 ångström.

Si l'on passe à l'arc dans l'air, on trouve les raies notablement élargies et cet élargissement s'accentue avec l'intensité du courant. Mais les diverses raies se comportent de façons très différentes quant au mode d'élargissement. Pour beaucoup de raies, l'élargissement est symétrique par rapport à la position de la raie fine, et il n'est pas très grand. Ce sont des raies relativement fines dans l'arc à la pression atmosphérique, du moins lorsque le courant n'est pas trop intense; leur largeur est alors d'environ 0.06 ångström. Quand on passe du vide à l'air on ne trouve, comme déplacement, que le petit accroissement de longueur d'onde dû à l'augmentation de pression (0.002 à 0.003 angström). Lorsque ces raies peuvent se renverser dans le spectre de l'arc, la raie d'absorption occupe le milieu de la raie d'émission. Elles constituent le groupe des *raies à élargissement symétrique.*

Pour d'autres raies, l'élargissement, quelle qu'en soit la cause, se fait d'une manière dissymétrique, et il est plus grand que pour les raies du groupe précédent; cet effet est déja notable à la pression atmosphérique, surtout lorsque le courant est intense. Pour certaines raies l'élargissement est assez grand, dans ces conditions, pour que leur aspect les distingue nettement dans un spectre de réseau, alors que, dans l'arc dans le vide, elles ne présentent rien de particulier.

Il est évident que cet élargissement dissymétrique produit un

déplacement de la raie dans le sens où l'élargissement est le plus grand; la position apparente de ces raies dépend donc de leur largeur, et par suite des conditions dans lesquelles on les produit. Lorsqu'on passe du vide à la pression atmosphérique, il y a non seulement le petit déplacement dû à la pression, mais encore l'effet de l'élargissement dissymétrique, qui est beaucoup plus grand et masque complètement le premier. D'autre part, dans l'arc à la pression atmosphérique, ces raies ne sont pas parfaitement fixes: elles subissent un déplacement apparent à mesure que l'intensité du courant augmente, et dans certains cas le déplacement est assez grand pour être constaté visuellement dans un spectre de réseau.

Les raies à élargissement dissymétrique forment deux groupes: (1) raies à élargissement vers le rouge; (2) raies à élargissement vers le violet.

Il est bien remarquable que certaines raies aient ainsi une tendance déterminée à s'élargir d'un seul côté, et cela sous l'action de causes en apparence aussi différentes que l'augmentation de pression et l'accroissement d'intensité de courant.

Pour un certain nombre de raies, nous avons mesuré par interférences, suivant la méthode décrite ci-dessus, la variation de longueur d'onde lorsqu'on passe de l'arc dans le vide à l'arc dans l'air; dans ce dernier cas, l'intensité du courant était faible (3 ampères environ), pour être dans les meilleures conditions de finesse des raies. Pour les raies à élargissement dissymétrique, les déplacements observés auraient été beaucoup plus grands si l'intensité du courant avait été plus forte.

Le tableau II (p. 112) donne le résultat de ces mesures; les raies sont réparties d'après le groupe auquel elles appartiennent.

Notre classification des raies a une certaine ressemblance avec celle que Duffield a déduite de ses expériences sur le déplacement par la pression: toutes les raies à élargissement vers le rouge appartiennent au groupe III de Duffield, qui est caractérisé par un grand déplacement sous l'action de la pression. Les raies à élargissement vers le violet ne se trouvent pas parmi celles que l'on a étudiées au point de vue de l'action de la pression; ces raies subissent, pour la variation de pression de 1 atmosphère lorsqu'on passe du vide à la pression atmosphérique, un déplacement apparent vers le violet;

il serait très intéressant de savoir comment elles se comportent aux pressions élevées.

Explication des anomalies observées dans la comparaison du spectre solaire avec celui de l'arc.—Dans le spectre solaire, ces différences d'aspect ne se manifestent aucunement. On n'aperçoit aucune dissymétrie dans l'absorption; les raies diffèrent uniquement par leur intensité, ou, ce qui revient au même, par leur largeur. Certaines raies qui, dans l'arc à la pression atmosphérique, sont nettement diffuses et se distinguent des autres au premier coup d'œil, ne présentent rien de particulier dans le spectre solaire. On peut citer, comme exemple

TABLEAU II

Longueurs d'onde (système internat.)	$\lambda_{\text{arc air}} - \lambda_{\text{arc vide}}$	$\lambda_{\odot} - \lambda_{\text{arc air}}$	Groupe
4181.76	+ 2	+ 2	Raies à élargissement symétrique
4315.09	+ 4		
5434.53	+ 1	+ 7	
4187.04	+11	− 3	Raies à élargissement vers le rouge
4191.44	+10	− 3	
4227.44	+20	−29	
4233.61	+12	− 8	
4235.95	+11	−19	
4250.12	+13	−14	
4859.75	+17	− 7	
4871.34	+10	−15	
5415.22	−15	+27	Raies à élargissement vers le violet
5424.09	−17	+30	

frappant de ce fait, les raies 5410.92, 5415.22, 5424.09. Il semble donc que la cause qui agit pour produire l'élargissement dissymétrique des raies d'émission ne se fait pas sentir sur les raies d'absorption. La raie d'absorption doit donc correspondre à la position qu'occupe la raie d'émission lorsqu'elle est rendue fine. Dans l'arc au fer, les raies à élargissement dissymétrique ne se renversent généralement pas.

Si l'on compare le spectre d'absorption du soleil avec le spectre d'émission de l'arc à la pression atmosphérique, on obtient la somme de deux effets: déplacement par la pression, et élargissement dissymétrique de certaines raies. L'existence de cette dernière cause explique complètement les résultats, en apparence incohérents, que l'on obtient en comparant le soleil à l'arc dans l'air. En effet:

Toutes les raies du groupe à élargissement symétrique donnent lorsqu'on passe de l'arc au soleil, un léger accroissement de longueur d'onde.

Toutes les raies du groupe à élargissement vers le rouge donnent une diminution de longueur d'onde.

Toutes les raies du groupe à élargissement vers le violet donnent une forte augmentation de longueur d'onde.

Ces résultats sont visibles, soit sur le tableau II où l'on a indiqué dans la 3^{me} colonne la valeur du déplacement en passant de l'arc au soleil, soit sur le tableau I, où l'on a indiqué dans la 5^{me} colonne le groupe auquel appartient chaque raie: *S* désigne les raies du groupe symétrique, *DV* les raies qui s'élargissent vers le violet, et *DR* celles qui s'élargissent vers le rouge.

Si, au lieu de partir de l'arc dans l'air, on compare directement le spectre solaire avec celui de l'arc dans le vide, l'élargissement dissymétrique de la raie d'émission est supprimé, ou du moins très atténué; les anomalies résultant des dissymétries disparaissent et toutes les différences sont de même signe. Le tableau III donne, pour quelques raies, le résultat de ces comparaisons.

TABLEAU III

Longueur d'onde (système internat.)	$\lambda_{\odot} - \lambda_{\substack{\text{arc} \\ \text{vide}}}$
4181.76	+ 8
4187.04	+10
4191.44	+ 5
4222.22	+ 5
4227.44	+ 6
4233.61	+12
4235.95	+ 3
4250.12	+11

On pourrait calculer indirectement ces mêmes différences en partant des variations constatées lorsqu'on compare successivement l'arc dans l'air à l'arc dans le vide et au soleil. Mais, comme pour les raies à élargissement dissymétrique l'arc dans l'air ne donne pas des raies parfaitement fixes, ce calcul indirect ne peut donner, pour ces raies, une bien grande précision.

Pression de la couche renversante.—L'influence de l'élargissement dissymétrique des raies étant éliminée, les différences qui subsistent entre les longueurs d'onde de l'arc et du soleil sont attribuables à la

pression de la couche renversante, et peuvent servir à calculer cette pression. Il faut pour cela connaître le déplacement des diverses raies par la pression.

Pour les raies qui s'élargissent d'une manière dissymétrique, ce coefficient de pression, déduit de mesures faites sur le spectre d'émission, ne paraît pas avoir de sens précis. En particulier, Duffield trouve, pour les raies de son groupe III, un coefficient de pression très élevé. Mais ces raies ne se renversent pas; aux pressions élevées elles sont fortement élargies, dissymétriquement du côté du rouge; les pointés faits sur une pareille raie ne donnent aucune indication sur la position de la raie d'absorption, seule intéressante au point de vue de l'étude du spectre solaire. En fait, lorsqu'on passe de l'arc dans le vide au soleil, le déplacement de ces raies est de même ordre de grandeur que pour celles qui s'élargissent symétriquement.

On utilisera donc, pour le calcul de la pression, les raies du premier groupe. Comme les valeurs des déplacements ne sont connues qu'avec une précision relative faible, aussi bien le déplacement de l'arc au soleil que le déplacement des raies d'émission par la pression, il est rationnel d'opérer sur des moyennes.

Sur 22 raies entre 4000 et 4500 le déplacement, quand on passe de l'arc à la pression atmosphérique au soleil est en moyenne de 0.0062 ångström; pour ces mêmes raies, le déplacement moyen dû à la pression[1] est de 0.00145 ångström par atmosphère. Il en résulte, pour la pression de la couche renversante, 4.5 atmosphères au dessus de la pression atmosphérique.

Sur 10 raies entre 5100 et 5500 la différence moyenne entre le soleil et l'arc est 0.0103, et le déplacement moyen dû à la pression est de 0.0024 par atmosphère, ce qui donne une pression de 4.5 atmosphères au dessus de la pression atmosphérique.

Ces deux résultats bien concordants conduisent à cette conclusion que, dans la région de l'atmosphère solaire où se produit l'absorption par la vapeur de fer, la pression est de 5 à 6 atmosphères.

III. LARGEUR DES RAIES DU SPECTRE SOLAIRE

Les raies du spectre solaire ne sont pas seulement caractérisées par leurs longueurs d'onde. Pour chacune d'elles, les tables, celle de

[1] D'après les observations de Duffield (*loc. cit.*) et de Humphreys (*Astrophysical Journal*, **26**, 18, 1907).

Rowland par exemple, donnent un chiffre dans une colonne intitulée *intensité;* cette valeur, dont la définition est un peu vague, caractérise, en quelque sorte, la visibilité de la raie. Il paraît a priori assez difficile d'assigner un sens précis à cette comparaison de deux raies lorsqu'elles sont placées dans des régions très éloignées du spectre. On peut essayer d'obtenir une donnée numérique plus précise en mesurant la largeur de chaque raie.

L'examen direct du spectre solaire ne peut donner que des indications imparfaites sur la largeur des raies fines, dont la largeur ne dépasse pas beaucoup le pouvoir de définition du réseau employé. On peut obtenir cette largeur en produisant des interférences avec des différences de marche croissantes et cherchant la limite de visibilité.

Pour faire cette étude, nous avons employé le dispositif qui vient d'être décrit. On utilisait seulement la lumière du centre du disque solaire, en limitant l'ouverture de l'écran à un cercle de 2 mm de diamètre. L'image du soleil ayant 28 mm, l'élargissement parasite dû à l'effet Doppler-Fizeau est pratiquement éliminé. On opère avec des différences de marche croissantes, en employant l'interféromètre. Nous nous sommes limités à la région 4400, et à des observations photographiques. On a fait une série de poses photographiques, avec des différences de marche croissantes de 10 à 30 mm. Soit, pour une raie, Δ la différence de marche (double de l'épaisseur de l'appareil interférentiel) pour laquelle les interférences cessent d'être visibles. La largeur $d\lambda$ de la raie est alors donnée par l'équation

$$d\lambda = \frac{\lambda^2}{\Delta}$$

On trouve ainsi que, pour une même valeur de l'intensité donnée par Rowland, la largeur est très sensiblement constante. On peut donc faire une table donnant la largeur de la raie en fonction de l'intensité de Rowland. Le tableau IV (p. 116) donne, pour chaque intensité, la valeur de la différence de marche limite Δ, et la largeur $d\lambda$ exprimée en ångström.

Par la manière même dont on les a obtenues, les valeurs de la largeur pourraient être erronées par excès: le calcul suppose qu'aucune autre cause que la largeur des raies n'intervient pour fixer la limite d'interférence; les imperfections de l'appareil interférentiel et du

pectroscope peuvent y contribuer. Toutefois, nos mesures fixent l'ordre de grandeur des largeurs de raies, largeurs qui ne paraissent pas avoir été mesurées jusqu'ici.

TABLEAU IV

Intensité	Différence de marche limite	Largeur
1	28 mm	0.07 ångström
2	23	0.085
3	19	0.10
4	17	0.115
5	15	0.13
6	14	0.14
8	12	0.16

IV. COMPARAISON DES SPECTRES DU CENTRE ET DU BORD DU SOLEIL

Les spectres du centre et d'un point du bord du soleil présentent une notable différence dans la position des raies, phénomène purement cinématique dû à l'effet Doppler-Fizeau produit par la rotation de l'astre. Cette différence, qui dépend de la position du point sur le bord, s'annule quand il est au pôle; on peut aussi s'en affranchir en utilisant deux points diamétralement opposés. Ayant éliminé cette variation, on peut comparer les spectres à un point de vue purement physique. On trouve ainsi que les deux spectres diffèrent par plusieurs caractères.

1. Il y a un changement d'aspect pour certaines raies. Pour les fortes raies, la pénombre est affaiblie dans le spectre du bord. Parmi les autres raies, quelques-unes sont renforcées ou affaiblies au bord du disque, et le changement est en général de même sens que dans le spectre des taches.[1]

2. Halm a annoncé[2] que certaines raies subissent, du centre au bord, un léger accroissement de longueur d'onde; ses mesures ont été faites sur deux raies rouges du fer, en comparant leurs positions à celles de deux raies telluriques voisines. Il a trouvé un déplacement de 0.012 ångström.

La méthode employée par Halm n'est applicable qu'à quelques raies exceptionnelles, très voisines de raies telluriques. Notre méthode

[1] G. E. Hale and W. S. Adams, *Astrophysical Journal*, **25**, 300, 1907.

[2] *Astronomische Nachrichten*, **173**, 273, 1907.

interférentielle a l'avantage de permettre la mesure des déplacements d'une raie sans la rapporter à aucune raie voisine.

L'image du soleil tombe sur l'écran percé d'une ouverture circulaire de 2 mm de diamètre. En agissant légèrement sur le miroir C (fig. 1) on peut amener sur cette ouverture telle région que l'on veut du disque solaire.

L'influence de la rotation solaire s'élimine en prenant successivement les deux extrémités d'un diamètre de l'image solaire, et faisant la moyenne des résultats ainsi obtenus. Il y a évidemment avantage à prendre le diamètre polaire; les résultats des mesures faites sur les deux extrémités doivent alors être identiques, ce qui fournit une vérification.

Nous avons opéré par photographie: on produit successivement sur la même plaque les spectres avec interférences provenant d'abord d'un pôle, puis du centre, et enfin de l'autre pôle. Nous avons utilisé plusieurs clichés, obtenus avec des différences de marche de 5 mm et 10 mm. A cause de la petite ouverture nécessaire pour limiter une région définie du disque solaire, et d'autre part à cause du faible éclat du bord, le spectre est assez peu lumineux. Nous avons employé le spectroscope à prismes dont on a parlé plus haut. Les mesures étant assez délicates, nous n'avons pas fait une étude complète de toutes les raies solaires, et nous avons préféré étudier avec soin les déplacements d'un petit nombre de raies de la région 4400. Les mesures ont porté sur 14 raies de différents métaux, d'intensité faible ou modérée (de 2 à 6 dans l'échelle de Rowland).

Le tableau suivant (p. 118) donne, pour chacune des raies étudiées, l'intensité et la substance d'après Rowland, et la différence entre la longueur d'onde au bord et au centre, exprimée en millièmes d'ångström.

Ces résultats sont d'accord avec celui de Halm. Lorsqu'on passe du centre au bord, il y a un petit accroissement de longueur d'onde, qui, pour les raies étudiées, varie de 0.003 à 0.007 ångström. Exceptionnellement, les deux raies du vanadium ne montrent aucun déplacement.

Elargissement des raies dans le spectre du bord du disque solaire.— L'accroissement de longueur d'onde n'est pas seule modification que subissent les raies en passant du centre au bord du disque. Elles

subissent en outre un élargissement, qui se manifeste par une diminution de netteté des interférences, diminution déja visible sur les clichés ayant servi aux mesures de déplacements (différence de marche 10 mm). Nous avons étudié spécialement ce phénomène en faisant une série de photographies du spectre avec des différences de marche progressivement croissantes.

On trouve ainsi que, dans le spectre du bord, chaque raie est un peu élargie; cet élargissement paraît un peu variable d'une raie à l'autre; il est moyenne, dans la région 4400, de 0.010 ångström.

On voit que pour la plupart des raies on trouve, en passant du centre au bord du disque: (1) un déplacement vers le rouge de 0.005 ångström; (2) un élargissement de 0.010 ångström.

TABLEAU V

Longueurs d'onde (système internat.)	Intensité	Substance	$\lambda_{\text{bord}} - \lambda_{\text{centre}}$
4346.56	2	*Fe*	5 millièmes d'ångström
4348.94	2	*Fe*	4
4351.05	3	*Cr*	5
4375.93	6	*Fe*	4
4379.23	4	*Va*	0
4400.38	3	*Sc*	3
4406.64	2	*Va*	0
4422.57	3	*Fe, Y*	3
4435.68	4	*Ca*	4
4461.64	4	*Fe*	4
4468.49	5	*Ti*	7
4485.67	3	*Fe*	7
4496.85	3	*Cr*	6
4534.78	4	*Ti*	5

Ces deux résultats peuvent se résumer en un seul énoncé: la seule modification que subit la raie est un déplacement de son bord rouge, s'élevant à 0.010 ångström, l'autre bord ne variant pas. Exceptionnellement, et c'est le cas des deux raies du vanadium, l'élargissement est symétrique.

Le fait que de nouvelles radiations sont absorbées, uniquement sur le bord rouge de la raie, peut être attribué à une absorption par les couches profondes de l'atmosphère solaire, où la pression est plus élevée: au centre du disque cette absorption ne produirait qu'un effet insignifiant, tandis qu'au bord, où toutes les couches sont traversées obliquement par la lumière, l'effet en pourrait être sensible, sans préjudice de l'absorption par les couches à pression modérée, produi-

sant la partie fixe de la raie. Un accroissement de pression de 7 atmosphères suffit à expliquer le changement observé, ce qui correspond à douze atmosphères environ pour la pression de la couche profonde.

Il faut remarquer que l'image solaire de 28 mm dont nous disposons est bien petite pour utiliser une région définie avec une ouverture de 2 mm, que l'on ne pouvait pas réduire sans diminuer par trop l'intensité de la lumière. On n'a pas seulement la lumière provenant rigoureusement du bord, et on aurait probablement des différences plus marquées si l'on pouvait opérer dans des conditions plus favorables.

D'ailleurs, notre but n'était pas d'épuiser la question; nous n'avions ni le temps ni le matériel nécessaire pour cela. Nous voulions seulement, après avoir élaboré une méthode, montrer les résultats qu'on peut en tirer, et faire voir combien l'étude approfondie des phénomènes au laboratoire est nécessaire pour l'interprétation des mesures brutes obtenues dans les observations astrophysiques.

Université de Marseille
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ON A NEW MOUNTING FOR A CONCAVE GRATING

By ALBERT EAGLE

INTRODUCTORY

Notwithstanding the generally recognized pre-eminence of the concave grating for spectroscopic work of precision, this instrument has not yet taken the place it deserves in the everyday use of the ordinary spectroscopic laboratory. The reason for this is not hard to find. Not only is the Rowland mounting very expensive, but it requires an inordinate amount of room, frequently prohibitive, and moreover practically necessitates being used in a totally darkened room, which is often neither convenient nor possible.

It is with the object of drawing attention to the value of the concave grating not only for classical spectroscopic research, but also, when suitably mounted, for general use, that the present paper is undertaken.

An important need which is coming to be felt more and more by spectroscopists is that of examining faint spectra with instruments of high resolving power. For such work the loss of definition and resolution caused by a temperature-change in the grating during the long exposures necessary is very considerable, and may be fatal. How detrimental is a slight change of temperature of the grating during an exposure may be seen by noting that since the wave-length λ at any given point is proportional to the grating space b, we have $\frac{d\lambda}{\lambda}=\frac{db}{b}$. Now $\frac{db}{b}$ is equal to about 0.00002 for a change of 1° C. Hence if we have light of wave-length $\lambda=5000$ Å. U., $d\lambda=0.10$ Å. U. Now a good 4-inch (10 cm) grating with 15,000 lines to the inch (6000 per cm) can easily be made to resolve 0.13 Å. U. in this region in the first order, so that a temperature-change of 1° C. during the exposure will make it impossible to resolve lines less than 0.23 t.-m. apart, while in the third-order spectrum, the resolving power will be reduced to less than one-third. Moreover such a creep of the lines during an expo-

sure will make the accurate determination of wave-lengths from a superposed comparison spectrum impossible.

The seriousness of this temperature-change during long exposures may be seen from Mr. W. G. Duffield's paper on "The Effect of Pressure upon Arc Spectra,"[1] in which he states that he had to wait for several months before the temperature became steady enough to make his work possible. In the mounting to be described the temperature is kept steady by inclosing the grating in a heavily lagged box, which would be hardly possible with Rowland's form of mounting.

In view of the fact that a plane grating mounted as a Littrow spectrograph is coming to be regarded as the best and most convenient instrument for general use, it seems opportune to call attention to the fact that nearly all the advantages which this type of instrument possesses, together with others, may be obtained from a concave grating. This method of using a plane grating consists in placing a lens in front of it to render the incident light parallel, and rotating the grating until the diffracted light which is required returns along the line of incidence and is brought to a focus by the same lens. The advantages of this form of mounting are obvious: the space taken up is small, the instrument can be used in a lighted room, and, if it is desired to keep the temperature constant during long exposures, it is very easy to have the surrounding box lagged with some non-conductor of heat; or, if a still greater constancy of temperature is necessary, to place a thermostat inside.

This method of mounting gratings seems destined to find increasing favor among spectroscopists and some very large instruments on this principle have recently been constructed by Professor Hale on Mount Wilson in California. He secures constancy of temperature by making the axis of the instrument vertical, the slit and camera being at a convenient height above ground, while the grating is at the bottom of a well.

As against a concave grating mounted after Rowland, a plane grating used as above possesses the disadvantages of not giving a normal spectrum—though, as I shall show later, this is of slight conseqence—and also that such a spectrograph is of no use for the ultra-

[1] *Phil. Trans.*, A, **208**, 129, 1908.

violet, as the achromatic lens used in practice is always made of glass which will not transmit light of wave-length below about λ 3500.

The astigmatism possessed by a concave grating sometimes gives it an advantage over a plane grating. If, for instance, the spectrum of a very small source, such as a small spark, or a small region of a source, such as a sun-spot, be required with a plane grating it will be necessary to give width to enlargements made from the negative by the use of a cylindric lens or an up-and-down motion, either of which is liable to manufacture false lines out of specks, etc., on the plate. If a concave grating be employed and the whole of the slit be covered with a screen except that part on which the image of the required portion of the light source falls, width will be imparted to the spectrum by the astigmatism.

The method of mounting a concave grating about to be described consists in using it in a similar manner to that described above for plane gratings, save that no lens is employed. This method of using a concave grating was suggested independently, but the author has since found that it is not entirely new, Lord Rayleigh having informed him that he uses it in this manner himself, and possibly it has been so used by other spectroscopists and physicists. As, however, to my knowledge no actual mounting suitable for the continuous use of the grating in this manner has been published,[1] and as some of the points of the theory of the concave grating in this position do not appear to have received attenion, it seems worth while to describe the present mounting and the behavior of the grating when so used.

In order to secure the best definition with a concave grating it is necessary that the slit should be situated on a circle described on the radius of curvature of the grating as diameter, in which case all the spectral lines are focused on the same circle. The distance from the grating to the slit will accordingly vary considerably with the angle of incidence in the mounting suggested. Hence in using a grating in this manner it must be capable of considerable motion in the line of sight and also of rotation about a vertical axis. The only motion necessary for the camera face is one of rotation about a vertical axis through its center. It is clear that the angle of swing of the camera

[1] No reference to this method of using a concave grating is given in Kayser's *Handbuch der Spectroscopie.*

will need to be—theoretically—equal to the angle of incidence of the light on the grating.

DESCRIPTION OF MOUNTING

Fig. 1 represents a diagrammatic plan of the mounting. *G* is the grating-holder standing by means of leveling screws (not shown) on a worm-wheel *W* of 120 teeth. In order to prevent the grating-holder from falling over if accidentally knocked, it is fastened to the worm-wheel by means of a short length of spiral spring. The grating is kept up to the front of its holder by means of two light U-shaped springs which press against it from behind in the middle of the sides. A glass plate is slid down close in front of the grating when not in use to prevent tarnishing, and consequently it is not taken out of its holder. The worm-wheel *W* rests on a carriage *C* and may be rotated by means of a worm sliding on the long keywayed shaft *L*. The carriage *C* is mounted on rails *R* constituting a geometric slide, along which it is moved by means of the double-threaded screw *M* of half-inch (12.7 mm) pitch. Both this screw and the keywayed shaft *L* are connected through universal joints *U* with handles *H* and *K* each of which carries a divided head and a nut running on a screw of millimeter pitch and reading against a millimeter scale. The divided head

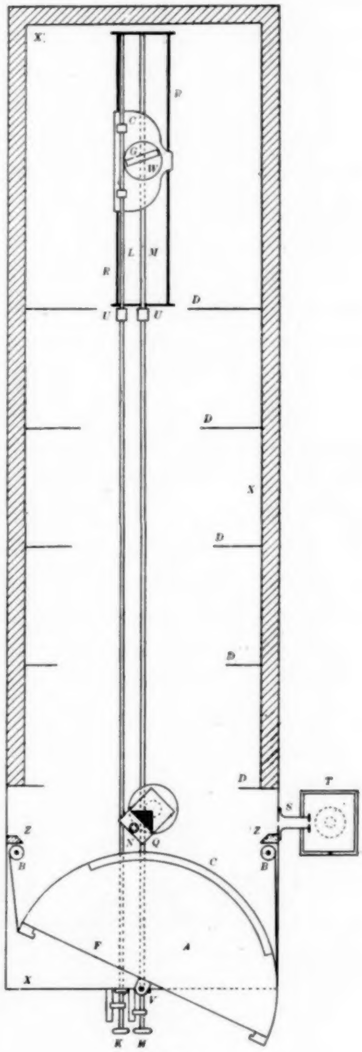


FIG. 1

which serves to rotate the grating is graduated into 100 parts, while the other one is divided into 10 parts.

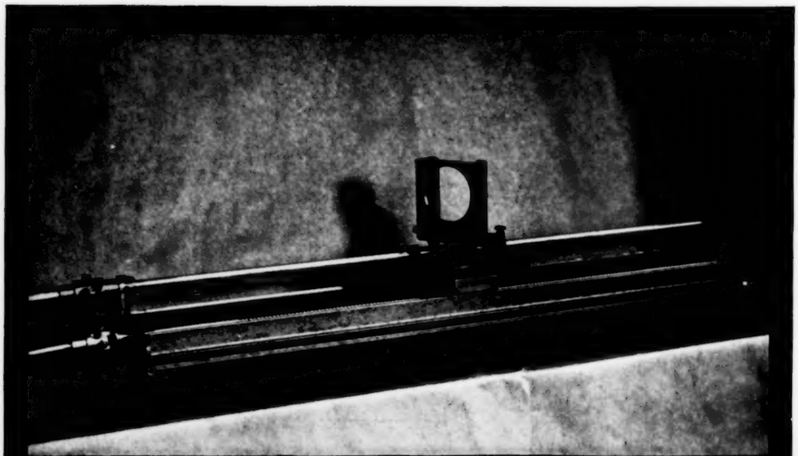
The surrounding box *X* is double-walled for nearly the whole of its length, the intervening space of 1½ inches (37 mm) being packed with slag wool. The outside dimensions of the box are: length 11 feet 1 inch (337 cm); breadth 25 inches (63 cm); and height 22 inches (56 cm). A long lid in the top of the box just above the rails gives access to the grating. A series of five diaphragms *D* are spaced between the grating and the camera; these prevent the light which reaches the sides of the box from the other orders from fogging the photographic plates. The apertures in the diaphragms are 4 inches (10 cm) high and of varying width so as not to interfere with the passage of the light. *F* is the camera face, capable of turning about two strong pivots *V*. The lower one is attached to a platform 10 cm above the bottom of the box. Below this platform pass the two rods which serve to move the grating. These pivots should lie as nearly as possible in the surface of the photographic plate.

Attached to the edges of the camera face are light-tight blinds which are kept taut by means of spring rollers *B*. Strips of wood *Z* are placed behind the rollers in order to prevent light from entering. Three semicircular discs around which the blinds work are fastened on the inner side of the camera face, the upper and lower of these work in contact with the top of the box and the platform respectively, while the third is midway between them and just above the slot in the camera face. Sufficient height must be left in the camera face above the slot to allow the shutter of the dark slide to be opened after it is in place. The upper semicircular disc carries a flat millimeter scale around its edge which is visible through a small window in the top of the box. A bolt also passes through the top, by means of which, with a hand nut, the camera can be tightly clamped in position. Two removable cheeks on the camera face enable it to take dark slides holding plates either 18×3 inches (45×7.5 cm) or $12 \times 2\frac{1}{2}$ inches (30×6.3 cm). The dark slides have been constructed so as to hold the plates in the focal curve, especially thin plates about 1 mm thick being used which permit of the bending. The dark slides are supported in the camera by means of a catch at each end.

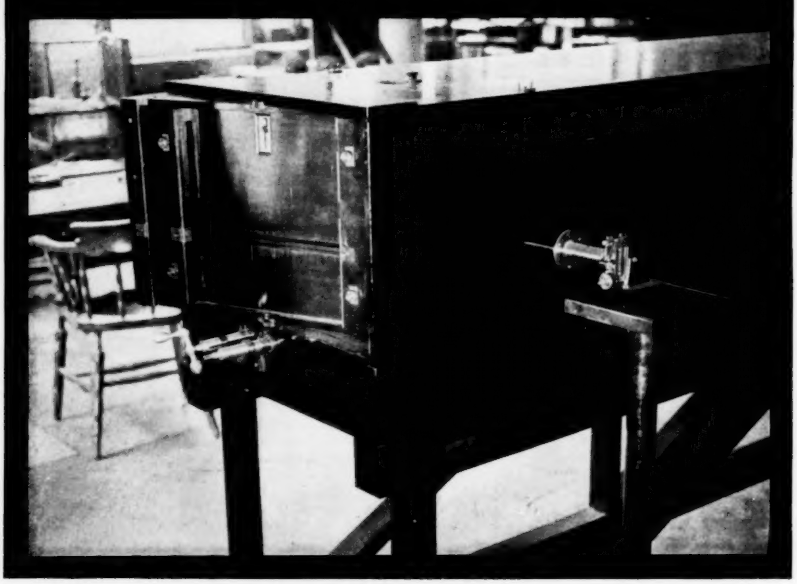
The slit is of Rowland's type and is placed in one side of the box

PLATE VII

1



2



1. Carriage for grating. 2. Camera end.

just below the central line of the grating. Light falls from the slit on a totally reflecting prism of quartz with faces $1\frac{1}{4}$ inches (3.2 cm) high and 1 inch (2.5 cm) broad. The distance of the slit from this prism should be nearly equal to the distance from the prism to the pivots on which the camera turns. The prism is cemented to a table carried on a pillar from the bottom of the box. By means of a leveling screw *N* reached by means of a hand-hole in the side of the box opposite to the slit, the table can be tilted about a horizontal axis parallel to the hypothenuse of the prism. The table also permits of being rotated about a vertical axis.

Plate VII, No. 1, is a photograph of the carriage and rails. The length of the rails is 3 feet 6 inches (106 cm), giving a travel of 2 feet 6 inches (76 cm). Plate VII, No. 2, shows a general view of the camera end of the instrument. It is supported at a convenient height above the floor by a rigid wooden girder resting on concrete piers built on foundations quite independent of the foundations of the building. Outside the slit is a small table, also shown at *T* in Fig. 1, supported independently by a column from the floor. On this can be placed a comparison shutter or any absorbing solution.

As the astigmatism except in the first order is too great to permit the employment of a comparison shutter outside the slit, a second comparison shutter is provided inside the instrument just in front of the photographic plate. This is operated by means of a small knob which can be moved up and down a slot near the top of the camera face and rested on different notches. The shutter is of thin sheet ebonite and made as light as possible. It contains a slot 1 inch (2.5 cm) wide which permits an unobstructed view of the spectrum, and also one a quarter of an inch (6.3 mm) wide which can be placed in three positions differing in height by $\frac{3}{16}$ of an inch (5 mm). The shutter is also capable of completely closing the slot in the camera face to keep out dust when not in use.

It is of course possible that the use of such a comparison shutter might introduce a slight shift between a spectrum and its comparison, but in half a dozen pairs of spectra which were taken as a test no such shift could be detected. If such a shift occurred it could be determined and allowed for by taking another plate in which the two spectra are directly superposed without the use of the comparison shutter.

THE MICROMETER EYEPIECE

The instrument has been provided with a micrometer eyepiece mounted on a board which fits in the camera face like the dark slide. In using this eyepiece the camera is set square on. By means of this, visual observations of particular lines may very conveniently be made, or the distance between close lines measured when it is not desired to take a photograph. By turning each handle while looking in the eyepiece the whole visible spectrum may be readily brought under review and examined. With this eyepiece observations and also measurements of the Zeeman effect may very conveniently be made.

It can easily be proved that the value of one division on the head of the micrometer eyepiece has a constant value in wave-lengths for any position in the spectrum. Calling i the angle of incidence and θ that of diffraction which is nearly equal to i, we have

$$Nm\lambda = \sin i + \sin \theta\,,$$

where N is the number of rulings per cm and m is the order observed. Hence

$$\mathrm{N}md\lambda = \cos \theta d\theta = \cos i d\theta\,.$$

If r be the distance from the eyepiece to the grating and ds be the apparent distance between two lines in the eyepiece, we have

$$d\theta = \frac{ds}{r}\,.$$

Now $r = R \cos i$, where R is the radius of curvature of the grating. Hence we obtain

$$ds = RNmd\lambda\,,$$

showing that the scale in the eyepiece $\dfrac{ds}{d\lambda}$ is constant throughout a given order.

By the use of this eyepiece the difference of wave-length between close lines may be measured with a probable error of only two or three hundredths of an Ångström in the first order

Curves have been constructed from which the wave-length of a line in the eyepiece may be determined to an Ångström or two from the reading on the head which serves to rotate the grating. A second curve on the same diagram shows the position of the other head for which the spectrum is in focus.

ADJUSTMENTS AND FOCUSING

An arc lamp is set up at a distance of some feet from the slit and in such a position that the light entering the slit is horizontal and at right angles to the side of the box. A lens is then introduced to throw an image of the arc on the slit. It has been found convenient to have this image lens, which is an achromatic glass one of $2\frac{1}{2}$ inches (6.3 cm) aperture and 12 inches (30 cm) focal length, mounted on rails parallel to the path of the light and the arc lamp on rails at right angles to this. For ultra-violet work a concave speculum metal mirror of $2\frac{1}{4}$ inches (5.7 cm) aperture and 9 inches (23 cm) focal length is employed, as a quartz lens does not produce an achromatic image. This is also mounted on rails, and in using it the arc is placed sufficiently out of the line of collimation to prevent the direct light from the arc which enters the slit from falling upon the grating.

The height of the lower edge of the slot in the camera face should be the same as that of the center of the grating, and the top of the quartz prism should be slightly below this so as to permit of an unobstructed view of the grating from the slot. Care must be taken when the rails and carriage are fixed in the box that the axis about which the grating rotates is vertical. This can be tested by means of a small spirit-level placed directly on the worm-wheel.

The position of the image-lens having been fixed, the totally reflecting quartz prism is adjusted by tilting and rotating about the vertical so that the reflected beam of light covers the grating symmetrically. The grating must now be leveled so that the spectrum is the right height in the field in all orders. This is readily done by first adjusting the two leveling screws behind the grating so that the spectra on each side of the normal are the same height in the eyepiece, and then adjusting the leveling screw in front till one of them is the correct height.

The instrument is focused for photography as follows. The handle K is turned till the required region is in the field of view. The grating is then advanced by means of the other handle till it is seen on observing the lines with an eyepiece that they are focused in approximately the right position in the center of the field. From the reading on K when the central image is brought into the center of the field of view, the inclination of the grating to the incident light can be found, the number of teeth on the worm-wheel being known. The camera

is then set to this inclination by means of its scale, the required scale-reading being given by adding the reading for no swing to the product of the inclination of the grating in radians into the radius of the scale. The final focusing must now be done by photography. A series of half a dozen photographs are taken on a single plate, the grating being advanced by $\frac{1}{20}$ of an inch (1.27 mm) between each successive exposure, the selected positions lying on each side of the approximate focus observed. The spectrum which shows the lines in the center of the field in best focus gives the position of the grating-carriage, while from the positions in which the ends of the plate are in focus the correction to be made to the swing can be worked out. It must be remembered that the focus moves in and out twice as fast as the grating. If δ is the difference in focus between the center of the plate and its ends, $2a$ its length, and r the radius of the scale recording the camera-swing, the correction to be made to the scale reading is $\frac{r\delta}{a}$.

The three position readings together with the range of wave-length obtained on the plate are then recorded, and to photograph any spectrum in this region in future, it is only necessary to set the three recorded readings on their respective scales, when the instrument will be in focus. A series of focusing plates as above are taken throughout the different orders and the results tabulated, from which in a few seconds the instrument can be set in adjustment for any desired region. Hence the instrument is hardly less convenient in use than the Rowland mounting, while the labor of taking the focusing plates is no more than that of originally setting the latter in adjustment.

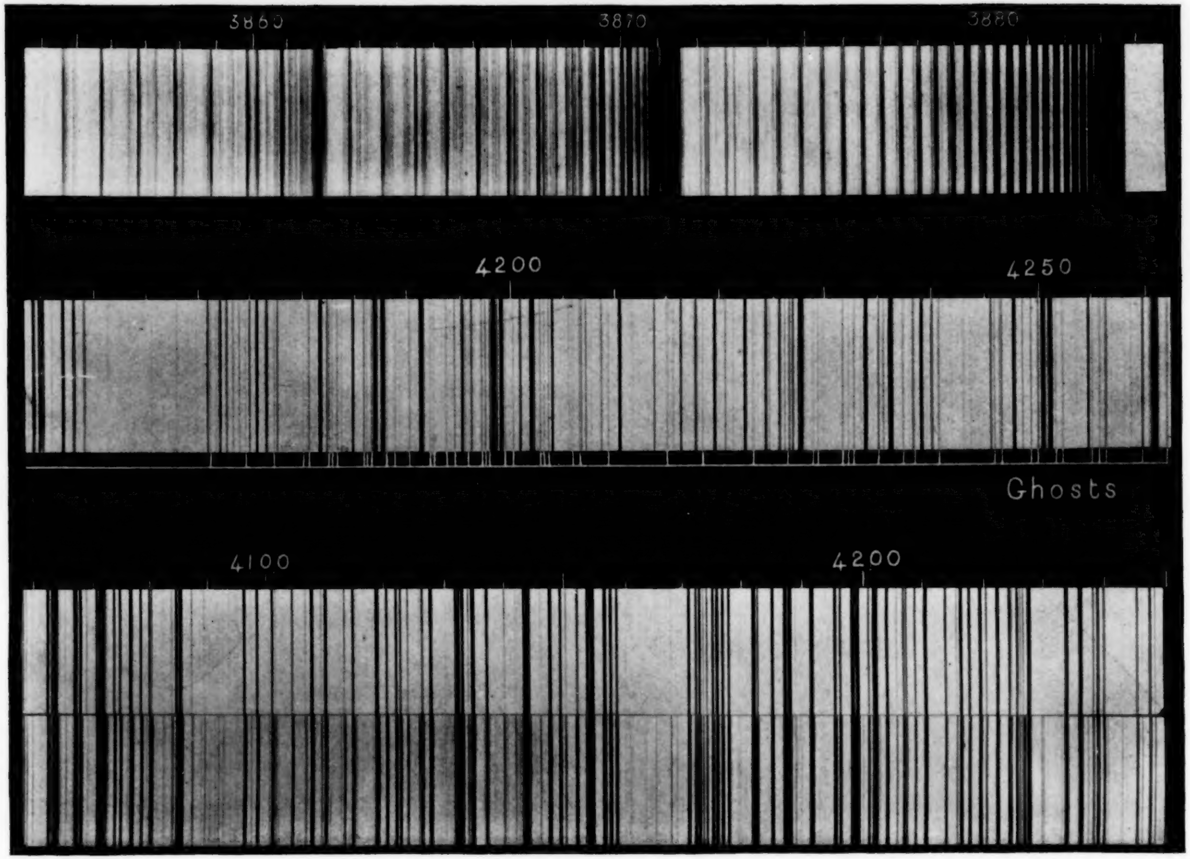
ADVANTAGES OF THE PRESENT MOUNTING

In comparison with Rowland's mounting the one under consideration has the following advantages:

1. The very small space occupied by it.
2. No darkened room is necessary.
3. It is very much cheaper than the Rowland mounting. Even when well made with all desirable additions the cost would be only about half that of the latter.
4. Spectra on either side of the normal may be used with equal

PLATE VIII

1
2
3
4



1. Head of CN band at λ 3883. Fifth order.
2. Iron arc. Fifth order.
3. Iron arc. Fifth order. Single exposure.
4. The same. Quadruple exposure.

facility, a point of some value, as it may happen that the best third-order spectrum is on the opposite side to the best first-order spectrum.

5. Everything being on the same axis, great rigidity is obtained. External vibrations would tend to shake the instrument as a whole rather than one part with respect to another, and consequently such vibrations would not affect the definition.

6. A slightly increased dispersion is obtained, especially in the higher orders. Referring to the section on the micrometer eyepiece it will be seen that the scale of the spectrum in the eyepiece $\frac{ds}{d\lambda}=RNm$. When this is projected on a plate inclined at an angle i the scale is clearly

$$\frac{RNm}{\cos i}.$$

The scale in Rowland's mounting is RNm throughout. No increase in theoretical resolving power is hereby obtained however.

7. Higher orders can be obtained than is possible with Rowland's mounting. Using the previous notation, if light be incident at an angle i, the wave-length diffracted along the normal will be

$$\lambda=\frac{\sin i}{Nm},$$

while the wave-length of the light diffracted back along the line of incidence will be

$$\lambda=\frac{2\sin i}{Nm}.$$

Hence the same wave-length can be obtained in this case with an angle of incidence of 30° as with grazing incidence in the former case. The mounting described is capable of accommodating light up to an angle of incidence of 40°, the limit being imposed by the length of the slide. This would have been made longer if it had been thought that the spectra obtained would be bright enough to be of any value, as they were found to be.

The utility of these higher orders may be seen from the photographs in Plate VIII. No. 1 shows the head of the well-known cyanogen band at λ 3883 taken in the fifth order with an exposure of 40 minutes. An absorbing solution of iodine in carbon bisulphide in a glass cell was placed before the slit. On the original negative lines only 0.05 Ångström apart are distinctly resolved.

8. The steadiness of temperature secured—a point of vital necessity in long exposures. Fig. 2 shows the variation of temperature inside and outside the box during a day of 9 hours. The true temperature inside was probably much more constant than that indicated by the thermometer which was inserted in a hole in the lid. Plate VIII, No. 3, shows a portion of the iron spectrum taken in the first order with an exposure of 40 seconds, adjacent to it being the same spectrum obtained with four superposed exposures of 10 seconds each, made at intervals of one hour. No special precautions were taken and other work was carried on in the laboratory as usual.

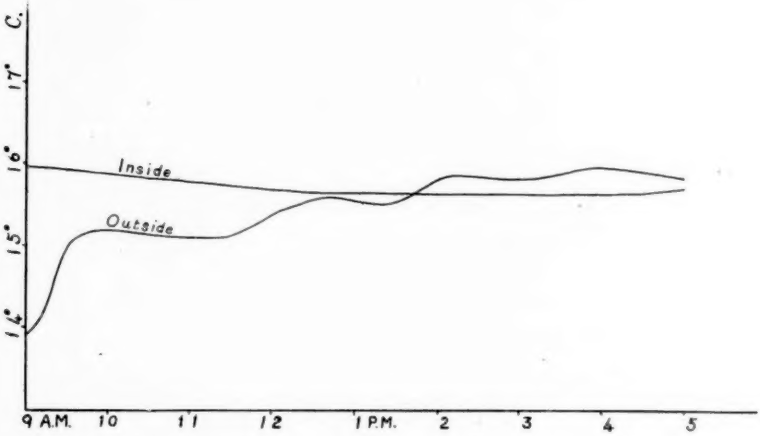


FIG. 2.—Temperature inside and outside of box

In a 28-hours exposure which was given on the hydrocarbon bands obtained from the flame of a Mecke burner, lines are resolved though only 0.15 Ångström apart. The resolving power for a short exposure would have been 0.11 Ångström (for a first-order spectrum), from which it can be calculated that the temperature must have remained effectively constant to 0.5° C. during the exposure, although the laboratory was not heated after 5 P. M. till 7 A. M. the next morning and the intervening night was frosty.

THE ASTIGMATISM OF A CONCAVE GRATING

One of the chief advantages of the present mounting is the great reduction in astigmatism which is effected, especially in the higher

orders which are thereby rendered much more brilliant. This reduction is so striking that it appears worth while to append proofs of the astigmatism in both cases, especially as no short proofs appear to have been given.

We will call the plane normal to the rulings at any point of the grating the principal plane of incidence at that point. If a pencil of light inclined at an angle α to this plane fall on the grating, it is clear that since there is no path-difference between rays falling on different portions of the same ruling all the diffracted pencils will also be inclined at an angle α to this plane, that is, they will lie on a cone of semi-vertical angle $\frac{\pi}{2}-\alpha$. The apparent angle β, between the incident pencil and the principal plane, when viewed in the direction of the intersection of this plane with the surface of the grating may easily be proved to be related to α by

$$\tan \beta = \frac{\tan \alpha}{\cos i},$$

where i is the angle of incidence projected on the principal plane. If α and β be small, we have $\alpha = \beta \cos i$.

Imagine a concave grating of radius R mounted in Rowland's manner and consider a pencil of light from a point at the center of the slit falling on the rulings at a height h above their center. We may regard this portion of the grating as part of a plane grating inclined at an angle $\frac{h}{R}$ to the vertical. In Fig. 3 let SA represent an incident ray and PA the trace of the principal plane of incidence at A, P being the center of curvature of the grating. Let AQ be the diffracted ray. The apparent inclination of the incident ray to the principal plane is

$$\beta = \frac{AC}{CS'} - \frac{AC}{CP}.$$

Now $CS' = GS \cos i = R \cos^2 i$, where i is the angle of incidence. Hence

$$\beta = \frac{h}{R}\left(\frac{1}{\cos^2 i} - 1\right) = \frac{h}{R} \tan^2 i .$$

α, the true inclination to the principal plane, is $\beta \cos i$. Hence

$$\alpha = \frac{h}{R} \tan i \sin i .$$

Since the diffracted ray is normal to the grating we have

$$\frac{QP}{AP}=\alpha\,.$$

Hence $QP=\alpha R=h\tan i\sin i$. If l be the total length of the rulings a point at the slit will therefore be drawn out into a line of length $l\tan i\sin i$ in the spectrum

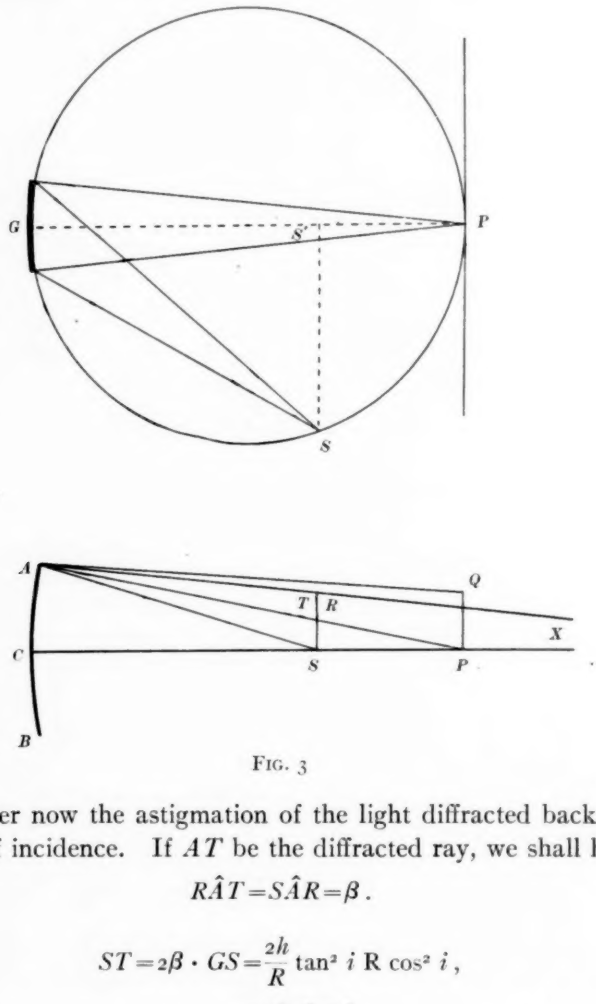


FIG. 3

Consider now the astigmation of the light diffracted back along the line of incidence. If AT be the diffracted ray, we shall have

$$R\hat{A}T=S\hat{A}R=\beta\,.$$

Hence

$$ST=2\beta\cdot GS=\frac{2h}{R}\tan^2 i\ \mathrm{R}\cos^2 i\,,$$
$$=2h\sin^2 i\,.$$

So the image of a point at the slit will be drawn out into a line of length $2l \sin^2 i$ in the spectrum.

Now in Rowland's mounting

$$\sin i = Nm\lambda$$

with the previous notation, hence the astigmatism is

$$\frac{N^2m^2\lambda^2 l}{\sqrt{1-N^2m^2\lambda^2}},$$

while in the present mounting

$$2 \sin i = Nm\lambda ,$$

and the astigmatism is consequently

$$\frac{N^2m^2\lambda^2 l}{2},$$

which, it is observed, is always less than half of the value in Rowland's mounting.

The following table gives a comparison of the angles of incidence and the astigmatism for the wave-length λ 5500 in the first five orders with a grating of 15,020 lines to the inch.

Order	Rowland's Mounting		Present Mounting	
	Angle of Incidence	Astigmatism	Angle of Incidence	Astigmatism
First	18° 59′	0.112l	9° 22′	0.053l
Second	40 35	0.557l	18 59	0.212l
Third	77 20	4.34l	29 12	0.476l
Fourth	Impossible	......	40 35	0.876l
Fifth	Impossible	......	54 23	1.321l
For $m\lambda = 16{,}911$	90 0	∞	30 0	0.5l

In view of the fact that the concave grating is sometimes discarded owing to the diminution of brightness caused by the astigmatism, it should be observed that so long as the length of slit illuminated exceeds the astigmatism, the brightness of the central portion of the spectrum is quite unaffected by it; but when it is less the brightness is directly proportional to the length of slit illuminated and inversely proportional to the astigmatism. Hence, as the above table shows, an enormous gain in brightness is secured in the higher orders with the present mounting as compared with Rowland's mounting. If the image-lens be arranged to magnify three times, it is generally possible to illuminate from 10 to 15 mm of the slit so that the whole of the first

two orders will be undiminished in brightness by the astigmatism. The brilliancy of one of the fourth-order spectra is such that at about λ 3800 a fairly heavy exposure of the iron arc may be obtained in three minutes.

As is well known, a stigmatic spectrum may be obtained with a concave grating if the screen instead of being close to the slit be placed at some distance in front of it. To find this position for the present mounting we observe that it is at the point X where AT produced intersects CP (Fig. 3).

Now

$$T\hat{X}S = A\hat{S}C - S\hat{A}X \,,$$

or

$$\frac{TS}{SX} = \frac{AC}{GS} - \frac{TS}{GS} \,;$$

that is,

$$\frac{2h \sin^2 i}{SX} = \frac{h}{R \cos i}(1 - 2 \sin^2 i) \,.$$

$$\therefore \quad SX = \frac{2R \sin^2 i \cos i}{1 - 2 \sin^2 i}$$

$$= 2\, Ri^2 \text{ approximately}$$

when i is small.

In the present grating the spectrum can be rendered stigmatic at λ 5500 in the first order by placing the screen 6.7 inches (17 cm) in front of the slit. It is not advisable to use the method when the distance much exceeds this, owing to the loss of light. The method is occasionally of value, when for instance the spectrum of different regions of a source is required. The image of the source is then focused, not on the slit, but the required distance in front of it. These distances necessary to obtain a stigmatic spectrum are approximately the same as in Rowland's mounting.

In view of the rather prevalent idea that the concave grating must be used in Rowland's manner in order to obtain the best definition[1] it may be pointed out that theoretically the definition is slightly more perfect in the present case than in Rowland's, though since the path-errors involved are too small to appreciably affect the definition in either case the point is not of much practical importance. It can be

[1] "Rowland's mounting combines the best definition with a normal spectrum."—Schuster, *Optics*, p. 122.

shown[1] that if light be incident at an angle θ on a concave grating of truly spherical form, the path-error of the light which falls on the sides of the grating over that which falls on its center is

$$\frac{b^4}{8R^3}\sin\theta\tan\theta\,,$$

where $2b$ is the breadth of the ruled space and R is the radius of curvature. In Rowland's mounting there is no path-error in the diffracted light since this converges to the center of curvature, hence the above expression is the total path-error. In the present case, since the light returns along the line of incidence, the path-error will be twice this expression, but since the same spectrum line is obtained with half the value of $\sin\theta$ and therefore with less than half the value of $\tan\theta$, it is seen that the path-error for the same line is less than half of what it is in Rowland's case.

DISADVANTAGES OF THE PRESENT MOUNTING

To set against the foregoing advantages of the present mounting over that of Rowland are the following disadvantages:

1. The instrument does not remain in automatic focus.

Once the series of focusing plates has been taken, however, the trouble of focusing the instrument again for any of these regions is practically nil, since we have only to set three known readings on their respective scales. If the regions of the focusing plates have been carefully chosen it will be seldom that a region not coinciding with one of them is required. If such be the case, either a special focusing plate may be taken for the region in question, or the scale-readings of the different focusing plates may be plotted on a large scale on millimeter paper and the required readings obtained from the curves. Another alternative is to find the corrections which have to be made to the positions of the nearest focusing plate to the region required by theory. This is readily done. If i be the inclination of the camera which is the same as that of the grating, the distance between them is $R\cos i$, while the wave-length at the center of the plate is

$$\lambda=\frac{2\sin i}{Nm}\,.$$

[1] Schuster, *ibid.*, pp. 121, 122.

Hence a change of δi in i will change the wave-length at the center of the plate by

$$\delta\lambda = \frac{2 \cos i}{Nm} \delta i \, ,$$

while the change in distance between the camera and the grating will need to be

$$R \sin i \; \delta i \, .$$

The angle i for the grating is readily obtained, knowing the previously determined reading for which the central image is reflected into the center of the field.

From the shift of wave-length which is required from the nearest standard position, δi is calculated and thence $R \sin i \; \delta i$, the required shift of the grating carriage. The value of i used in this formula should strictly be the value of i for a position midway between the region required and that of the nearest focusing plate With regions focused in this manner we have secured the same perfection of definition as in the standard positions.

2. Another disadvantage of the new mounting arises when photographic plates made on poor quality glass are employed, owing to the fact that the light is not incident normally on the plate. i being the angle of incidence, it is clear that a local depression of depth h will displace a line happening to fall in it by $h \tan i$. Such displacements are revealed by an irregular curve of errors when wave-lengths are determined. It is not difficult, however, to obtain plates which do not thus introduce sensible errors, and in one batch of plates which gave rise to them it was clear by looking obliquely over the surface that it was very irregular.

This is not a disadvantage which is peculiar to the present spectrograph since in many instruments in use the plate is very considerably inclined to the incident light, angles of over 60° being used in some cases.

3. A third disadvantage is that the spectrum obtained is not quite normal. This however is not so detrimental as might at first sight be supposed. The greatest deviation between the true wave-length and that calculated from a linear interpolation formula taken over a three-inch (7.6 cm) range in the green of the first order is only 0.2 Å. U. Such a deviation makes it generally practicable to employ a simple

linear interpolation formula between two standards and to draw a curve of errors showing the difference between the calculated and the true wave-lengths on intermediate standard lines From this curve the correction to be added to the unknown wave-lengths can be determined. In all cases of the determination of accurate wave-lengths, even when using a Rowland mounting, such a curve of errors should be constructed, if only to detect and eliminate accidental errors of setting on the standard lines from which the equation is calculated. So far, then, the fact that the spectrum is not normal makes no difference whatever.

When however it is required to obtain a uniform reduction over a much longer range it is best to use a formula of the type

$$\lambda = a + bs + cs^2,$$

which represents the spectrum with the same accuracy as it is represented by a linear formula in Rowland's case. Let λ_1, λ_2, λ_3 be the wave-lengths of three lines from which it is required to determine the constants of the above equation and s_1, s_2, s_3 their scale-readings, of which the second should be about midway between the first and third. The following solution is perhaps the most convenient for use:

$$\lambda = \left\{ a - \frac{e}{4}(s_1 - s_3)^2 \right\} + bs + e(s - s_m)^2$$

where

$$b = \frac{\lambda_1 - \lambda_3}{s_1 - s_3},$$

$$a = \lambda_1 - bs_1$$

$$e = \frac{\lambda_2 - a - bs_2}{(s_2 - s_1)(s_2 - s_3)},$$

and

$$s_m = \frac{s_1 + s_3}{2}.$$

In this form the constants are adapted to logarithmic calculation, while the last term, which is small, is of a very convenient form for evaluation with a slide-rule, which will generally be found sufficiently accurate. For the evaluation of the first two terms of the expression for λ, I am in the habit of using an arithmometer. With the above equation it will be seen that the extra trouble involved in not dealing with a normal spectrum is very small.

Even when this equation is used, a curve of errors should still be drawn as before by means of intermediate standards.

It may be shown that if the plate on which the spectrum is photographed be bent into a circle of radius R instead of into one of diameter R, the spectrum obtained will be as normal as that obtained with Rowland's mounting. Hence those who consider a normal spectrum a necessity cannot on that account alone reject the present mounting, as it may be obtained on a plate which neither requires so much bending as is necessary to fit the focal curve, nor goes out of focus as fast as a flat plate.

This great dependence of the law of dispersion in the spectrum upon the form of the plate may readily be shown as follows:

We have

$$\sin i+\sin (i+\theta)=Nm\lambda$$

where $i+\theta$ is the angle of diffraction.

Hence

$$\frac{d\theta}{d\lambda}=\frac{Nm}{\cos (i+\theta)}=\frac{Nm}{\cos i}\{1+\theta \tan i\}$$

$$=\frac{Nm}{\cos i}\left\{1+\frac{s}{R}\tan i\right\},$$

approximately, since, as the plate must very nearly fit the focal circle, θ must still be very nearly equal to $\frac{s}{R}$, where s is the distance of any line from the center of the plate; i. e., $s=0$ corresponds with $\theta=0$.

Now

$$\frac{ds}{d\lambda}=\frac{ds}{d\theta}\frac{d\theta}{d\lambda}.$$

The determination of $\frac{ds}{d\theta}$ is a purely geometrical problem. Let APB (Fig. 4) represent the photographic plate. Let $AP=s$, $OP=r$, and $P\hat{O}A=\theta$. Let the equation of APB in polar co-ordinates be

$$r=a+b\theta+c\theta^2 .$$

Then we have

$$\frac{ds}{d\theta}=\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{1}{2}}=\{a^2+b^2+(2ab+4bc)\theta\}^{\frac{1}{2}},$$

approximately. Write $a^2+b^2=A$ and $2ab+4bc=B$. Then

$$\frac{ds}{d\theta}=A^{\frac{1}{2}}+\frac{B}{2A^{\frac{1}{2}}}\theta ,$$

approximately. To a first approximation we have from this

$$s = A^{\frac{1}{2}}\theta \, .$$

Substituting this and also the values of A and B we have

$$\frac{ds}{d\theta} = \sqrt{a^2+b^2} \left\{ 1 + \frac{(ab+2bc)}{(a^2+b^2)^{\frac{3}{2}}} s \right\} \, .$$

The radius of curvature of the plate at A is found by means of the usual formula to be

$$\rho = \frac{(a^2+b^2)^{\frac{3}{2}}}{a^2-2ac+2b^2} \, .$$

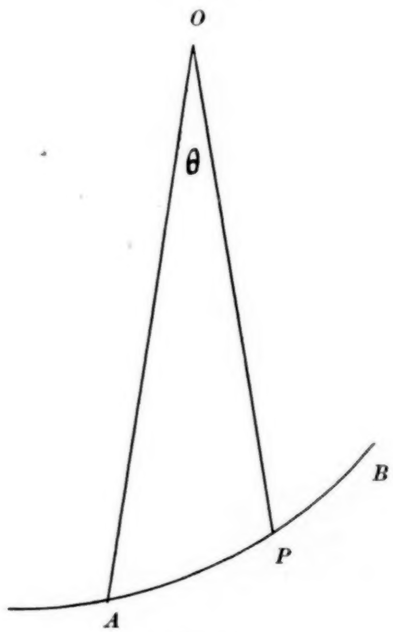


FIG. 4

Using this to eliminate c we obtain

$$\frac{ds}{d\theta} = \sqrt{a^2+b^2} \left\{ 1 - \frac{b}{a}\left(\frac{1}{\rho} - \frac{2}{(a^2+b^2)^{\frac{1}{2}}}\right) s \right\} \, .$$

Since the plate must very nearly coincide with the focal curve we may put $a = R \cos i$ and $\frac{b}{a} = -\tan i$. Then

$$\frac{ds}{d\theta} = R \left\{ 1 + s \tan i \left(\frac{1}{\rho} - \frac{2}{R}\right) \right\} \, .$$

Hence

$$\frac{ds}{d\lambda}=\frac{RNm}{\cos i}\left\{1+s\tan i\left(\frac{1}{\rho}-\frac{1}{R}\right)\right\}$$

or

$$\frac{d\lambda}{ds}=\frac{\cos i}{RNm}\left\{1-s\tan i\left(\frac{1}{\rho}-\frac{1}{R}\right)\right\}$$

to the order of approximation to which we have been working.

From this it will be seen that if $\rho=R$, $\frac{d\lambda}{ds}$ will be independent of the first power of s and therefore λ will be expressed in terms of s by an equation of the form

$$\lambda=a+bs+ds^3\ ,$$

showing that the spectrum is normal at $s=0$. We also note that the coefficient of s in the expression for $\frac{d\lambda}{ds}$ has equal values but of opposite signs when we make $\rho=\infty$ and $\rho=\frac{R}{2}$. That is, if we use plates to fit the focal curve the coefficient of s^2 in the equation

$$\lambda=a\ +\ bs+cs^2$$

will have the same value but the opposite sign that it has for a flat plate. This was discovered experimentally when curved plates were first substituted for the flat ones previously used.

In Rowland's mounting also, the law of dispersion is considerably influenced by the form of the plate; for instance, the spectrum will deviate from normal three times as much if a flat plate be employed as it will if a curved plate of the proper radius be used.

We have in this case

$$\sin\theta=Nm(\lambda-\lambda_n)\,,$$

where θ is the angle of incidence and λ_n is the wave-length diffracted along the normal. For a curved plate $\theta=\frac{S}{R}$ where s is the distance of any line from the center of the plate.

Hence

$$Nm(\lambda-\lambda_n)=\sin\frac{s}{R}=\frac{s}{R}-\frac{1}{6}\frac{s^3}{R^3}+\ .\ .\ .\ .$$

If on the contrary a flat plate be used we have $s/R=\tan\theta$, and hence

$$Nm(\lambda-\lambda_n)=\frac{s/R}{(1+s^2/R^2)^{\frac{1}{2}}}=\frac{s}{R}-\frac{1}{2}\frac{s^3}{R^3}+\ .\ .\ .\ .$$

showing that the coefficient of s^3 is three times as great as in the former case.

If a spectrum be represented by the equation

$$\lambda = a + bs + cs^2 + ds^3$$

it can be shown that the deviation from a linear formula fitting lines at s_1 and s_2 is

$$\Delta\lambda = \{c + d(s + s_1 + s_2)\}(s - s_1)(s - s_2) .$$

From this it may be calculated that if an 18-inch (46 cm) curved plate be used with a 10-foot (305 cm) concave grating mounted in Rowland's manner, a linear formula used over a region near one end of the plate will give a departure of more than 0.01 Å. U. from the true wavelength if the range taken exceed 36 mm in length.

PERFORMANCE OF A CONCAVE GRATING WITH THE PRESENT MOUNTING

Attention has already been drawn to Plate VIII, No. 1, showing the head of the cyanogen band at λ 3883 taken in the fifth order as an illustration of the value of higher orders than can be obtained in Rowland's mounting. No. 2 on the same plate represents a portion of the iron spectrum taken in the other fifth-order spectrum.

The resolving power secured in long exposures has also been mentioned, and it may be noted here that in the case of the four superposed exposures made at intervals of one hour the close double 0.118 Ångström wide, at λ 4240, is seen to be resolved at the tips of the lines, on the negative. This double is always resolved in the first-order spectrum unless overexposed or a specially wide slit be employed. This gives a realized resolving power of 36,000, whereas the theoretical purity with the width of slit employed (0.012 mm) is only 29,000. This fact, that a higher resolving power can be obtained than the theoretical value given by the ordinary formula, has been pointed out by other observers.

In the fifth-order spectrum lines only 0.05 Å. U. apart can be resolved. This, although greater than the theoretical value, which is about 0.03, seems to be limited by the accuracy of the rulings of the grating; 0.05 seems however to be about as small as Rowland was able to resolve in his largest gratings. Possibly also the majority of spectrum lines are not homogeneous to a much higher order than this.

The present mode of mounting and using a concave grating is not merely put forward as a possible alternative to Rowland's manner of using it, but I venture to suggest it as the most convenient and suitable mounting for all purposes and one which adapts the concave grating to the uses of general spectroscopy while sacrificing nothing of the high resolving power and excellent definition which make it suitable for the most refined research. The present mounting makes a compact spectrograph capable of dealing not only with the visible spectrum like a plane grating mounted in the Littrow manner but with the whole of the ultra-violet also.

The suggested mounting would be even more advantageous for a 21-foot (640 cm) grating than it is for a 10-foot (305 cm) one.

The mounting as above described has been constructed for the Spectroscopic Laboratory of the Imperial College of Science and Technology. The author's thanks are due to Professor Fowler for consenting to mount the grating in this manner, and also to Professor Callendar for the facilities he has afforded for having it entirely constructed in the workshop of the college under my supervision. Finally the author's thanks are due to Mr. W. Colebrook, the chief mechanician of the workshop, for the care and attention he has paid to the whole of the work, and also for his valuable suggestions on points of construction which presented difficulties, to the successful overcoming of which the perfection and success of the present instrument is largely due.

Imperial College of Science and Technology
London
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THE ABSOLUTE WAVE-LENGTHS OF THE H AND K LINES OF CALCIUM IN SOME TERRESTRIAL SOURCES[1]

By CHARLES E. ST. JOHN

The part taken by the H and K lines of calcium in solar and stellar spectroscopic theory is of such importance that all available knowledge of them is valuable, whether it concerns their wave-lengths, their individual characteristics, or their varying appearance under different conditions. The purpose of the present investigation is the determination, with the accuracy obtainable through the use of gratings of high dispersion, of the wave-lengths of the H and K lines referred to secondary standards based upon the red cadmium line adopted as the primary standard by the International Union for Co-operation in Solar Research at Meudon in 1907. The work is preliminary to a comparative study of the corresponding solar lines. For such an application of the results it is desirable that the wave-lengths in terrestrial sources should be known with the highest obtainable accuracy. The wave-length of the cadmium standard has been fixed as 6438.496 Ångströms, which defines the Ångström as equal to 10^{-10} m with an accuracy of one part in ten million. In the following paper wave-lengths based on this system are indicated by the subscript A.

	H	K
Rowland (arc)*	3968.617	3933.809
Kayser and Runge (arc)†	3968.63	3933.83
Jewell (arc)‡	3968.603	3933.794
Adams (arc)§	3968.629	3933.818
Exner and Haschek (spark)†	3968.62	3933.63
Eder and Valenta (spark)†	3968.638	3933.803
Cooper (spark)¶	λ_A 3968.488	λ_A 3933.686

* *Astronomy and Astrophysics*, **12**, 332, 1893.

† Eder und Valenta, *Beiträge zur Photochemie und Spectralanalyse* (Vienna, 1904), p. 336.

‡ *Astrophysical Journal*, **3**, 112, 1896.

§ *Contributions from the Mount Wilson Solar Observatory*, No. 6; *Astrophysical Journal*, **23**, 45 1906.

¶ *Astrophysical Journal*, **29**, 332, 1909. In a personal letter Dr. Cooper corrects an error in the published value of the wave-length of K. His corrected value is given above.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 44.

The differences in the values assigned to the wave-lengths of the H and K lines of calcium by different observers probably depend in great part on the standards involved. Some representative determinations are shown in the table on p. 143.

In the case of the determinations by Rowland, Jewell, and Adams, $H-K=34.808$, 34.809, 34.811, respectively. This close agreement in the relative values for H and K indicates a high degree of accuracy in the separate determinations and makes it probable that the absolute differences are to be referred to the different standards used. In view of these discrepancies, it was necessary to redetermine the wave-lengths in terms of accepted standards which can also be applied to the solar lines. The recently published absolute wave-lengths for iron lines by Fabry and Buisson,[1] prepared in accordance with the resolution adopted at the Meudon meeting of the International Solar Union,[2] furnished the standards used. The five iron lines $\lambda\lambda$ 4021, 3977, 3935, 3906, and 3865 include the desired region and are made the basis of the following determinations in arc, spark, and electric furnace.

APPARATUS

Three entirely different instruments, all spectrographs of the Littrow form, were used and served as checks upon each other in determining the wave-lengths of H and K.

1. The 13-foot spectrograph in the Pasadena laboratory of the Mount Wilson Solar Observatory.[3] This consists of a 5-inch (127 mm) objective of 13 feet (4 m) focal length and a Rowland plane grating having 14,438 lines to the inch (568 lines to the cm) placed in a deep dry well. It was used in the third order where the scale is approximately 1 mm = 1.38 Å.

2. The 18-foot spectrograph used in connection with the Snow horizontal telescope and mounted above the 5-foot spectroheliograph. In this spectrograph was used an 8-inch (202 mm) Michelson grating having 15,000 lines to the inch (590 lines to the mm), and a 6-inch (152 mm) lens of 18 feet (5.5 m) focus. The grating gave excellent

[1] *Astrophysical Journal*, **22**, 169, 1908.

[2] *Transactions of the International Union for Co-operation in Solar Research*, **2**.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 27; *Astrophysical Journal*, **28**, 244, 1908.

definition in the third order, though having a small amount of astigmatism due to a ruling error. The third order was used in the following investigation, the scale of which is very nearly 1 mm = 1.02 Å.

3. The 30-foot spectrograph of the tower telescope.[1] In this a Rowland plane grating, class A, with 14,438 lines to the inch was used, placed at the bottom of a deep dry pit of practically constant temperature. The performance of this grating is always excellent. The third order was used, the scale being about 1 mm = 0.60 Å.

CALCIUM IN THE ARC

The source of light for this part of the investigation was an electric arc between two iron poles or between iron and carbon poles when the absorption lines of H and K were to be measured, and between copper poles when it was desired to use the emission lines of H and K. Metallic calcium or a dilute solution of calcium chloride was introduced into the arc according as absorption or emission lines were desired. With large amounts of calcium vapor in the arc the appearance of these lines is familiar showing fine sharp reversals in the middle of the bright emission lines. The majority of the measurements were made upon the reversals, but some were made on the fine sharp symmetrical bright lines produced with very small quantities of calcium in the arc.

In the case of the reversals, the calcium being in the iron arc, the comparison spectrum was photographed simultaneously with that of calcium, and there can be no question of any disturbance due to a movement of any part of the apparatus. Two of the iron lines have very nearly the same wave-length as H and K, respectively, and partially overlap and tend to broaden them on the violet side when the calcium lines are very narrow. As it was desired to compare the wave-lengths for large and very small amounts of calcium in the arc, a copper arc with extremely small quantities of calcium was used for obtaining the narrow emission lines, the iron comparison spectrum being introduced by means of an occulting bar and spanning the narrow calcium spectrum. The agreement of the wave-lengths for the two classes of lines indicates no instrumental shift between the calcium and iron spectrum.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 23; *Astrophysical Journal*, **27**, 204, 1908.

The plates were measured on a Gaertner machine, with red right and red left, and in all cases, at least four settings were made on each line, the cross-hair being brought alternately from the right and left to the middle of the line. In the reduction of the measurements the factors were obtained for each of the four intervals given by the five standards and plotted as ordinates with the mean wave-lengths of the corresponding intervals as abscissae. These gave uniformly smooth curves with which the points agreed well even when a large scale was used. The standards proved perfectly satisfactory in this regard. In the calculation of the wave-lengths, H was determined from its distance from λ 3977 and λ 3935, and K from its distance from λ 3977, λ 3935, and λ 3906. Two iron lines, $\lambda\lambda$ 3948 and 3930, were also measured on each plate as a control, and in the case of λ 3930 its absolute wave-length was used in the later investigation. The factors can be read from the curve to the fourth decimal place directly and to the fifth approximately; this degree of accuracy is necessary for a run of 40 Å, as in determining K from λ 3977. The factor-curve for plate No. 521, which is an average example, is shown herewith.

The results for each line, as calculated from this curve, are given below.

From	H	λ 3948	K	λ 3930
λ 3977	3968.475	3948.784	3933.666	3930.302
λ 3935	.476	.783	.672	.297
λ 3906			.667	.298
	3968.476	3948.784	3933.668	3930.299

The best standard wave-lengths hitherto determined are those of Kayser's iron lines.[1] To compare the internal agreement of the two systems, the points for the factor-curve of plate No. 521 were obtained by using Kayser's wave-lengths for the corresponding Fabry and Buisson standards and were plotted as crosses on the same sheet. The curve for these points would be quite different from that shown, and owing to its irregularities the reduction factors could not be determined from it with the same accuracy. The close agreement between the values for H and K determined from the Fabry and Buisson

[1] *Astrophysical Journal*, **12**, 329, 1901.

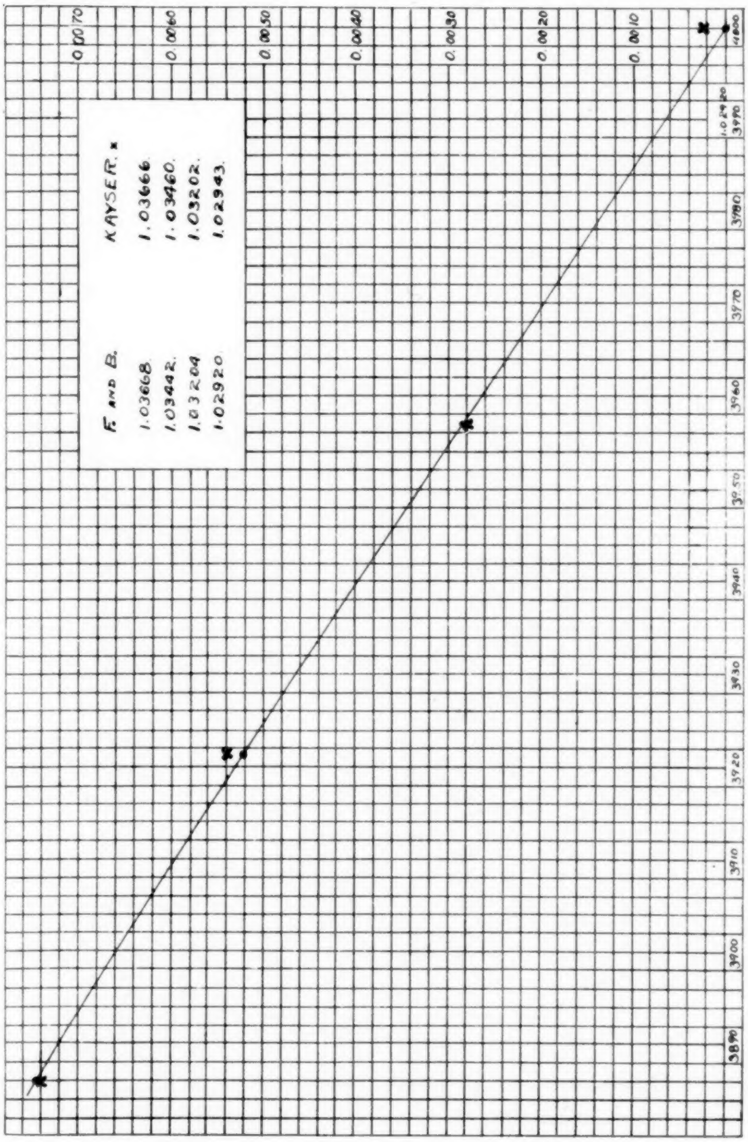


Fig. 1.—Factor-Curve for Plate No. 521

standards in the case of a single plate, depends upon the accuracy with which the factors could be interpolated from the curve.

Table I contains the results from the twenty-five spectra selected for measurement.

TABLE I

Plate	H	λ 3948	K	λ 3930	Spectrograph
348	3968.475 r	3948.784	3933.667 r	3930.300 r	18-foot
349	.476 r	.783	.668 r	.300 r	"
420	.476 r	.785	.667 r	.297 r	"
422	.477 r	.785	.666 r	.297 r	"
423	.478 r	.783	.667 r	.297 r	"
432 (1)	.473 r	.784	.668 r	.300 r	30-foot
(2)	.472 r	.781	.665 r	.302 r	"
(3)	.474 r	.782	.664 r	.300 r	"
476 (1)	.478 r	.787	.667 r	.299 r	18-foot
(2)	.479 r	.784	.665 r	.301 r	"
479	.475 r	.783	.667 r	.302 r	"
480 (1)	.478 r	.784	.665 r	.302 r	"
(2)	.478 r	.784	.665 r	.302 r	"
519 (1)	.479 r	.784	.667 r	.301 r	"
(2)	.475 r	.786	.669 r	.299 r	"
520 (1)	.478 r	.782	.671 r	.303 r	"
(2)	.478 r	.783	.670 r	.301 r	"
521 (1)	.476 r	.784	.668 r	.299 r	"
(2)	.475 r	.784	.668 r	.300 r	"
522 (1)	.473 r	.782	.667 r	.300 r	"
(2)	.473 r	.783	.666 r	.303 r	"
523 (1)	.476 r	.784	.666 r	.304 r	"
(2)	.476 r	.784	.667 r	.301 r	"
B 44 (1)	.473 b	.780	.667 b	.302 r	30-foot
(2)	.475 b	.780	.667 b	.302 r	"
Means of all	3968.476	3948.783	3933.667	3930.301	
Mean residuals	1.7	1.3	1.1	1.5	
Means, 18-foot	3968.476	3948.784	3933.667	3930.300	
Means, 30-foot	3968.473	3948.781	3933.666	3930.301	
Means, reversals	3968.476	3948.783	3933.667	3930 301	
Means, emission	3968.474	3948.780	3933.667	———	

In the above table the reversed and bright lines are indicated by the letters "r" and "b," respectively. An inspection of the table shows the close agreement between the results obtained with the two spectrographs, and the practically identical wave-lengths for both the reversed and bright lines of calcium. The K line was found to be more satisfactory for measurement than H; as an absorption line it is sharper and cleaner, when narrow, than the H line. The residuals represent fairly the difference between the two lines in this particular.

To compare these results with those found previously it is neces-

sary to express them in the Rowland-Kayser system. The reduction factor was obtained by using Kayser's wave-lengths.[1]

λ_A	λ_K	$\lambda_K \div \lambda_A$
4021.872	4022.029	1.000390
3977.745	3977.892	370
3935.818	3935.966	376
3865.526	3865.670	373
Mean		1.000377

Using this factor the following equivalents were found: The H line λ_A 3968.476 = λ_K 3968.625 and the K line λ_A 3933.667 = λ_K 3933.815. H − K = 34.810, which agrees closely with the mean difference 34.809 obtained from the Rowland-Jewell-Adams determinations, while the wave-lengths of H and K differ by 0.004 and 0.003 Å, respectively, from those found by Adams.

The standards used are known for only one of the observers, namely Adams. With respect to these Adams says:

> The first four plates were reduced with Rowland's values for the aluminium lines and λ 3973, and Kayser's value for the iron line λ 3928. The last eight plates were reduced with the use of Kayser's iron standards wholly. . . . . That the choice of standards may vitally affect the results is evident from the difference in wave-length shown by 0.013 tenth-meters assigned by Kayser and Rowland to the iron line λ 3928.

For the Kayser iron standards the internal disagreement is sufficient to account for the difference between Adams and the writer, as shown below.

λ_A	λ_K	$\lambda_K \div \lambda_A$
3927.921	3928.073	1.000387
3935.818	3935.966	376
3966.068	3966.219	381
3969.261	3969.411	378
Mean		1.000380

From Mr. Adams it was learned that he used the wave-lengths of the following iron lines from Kayser's table in reducing his last eight plates, namely λλ 3969, 3966, 3935, and 3928 The line at λ 3935 is a Fabry and Buisson standard, and as the writer had

[1] *Loc. cit.*

determined the other lines in terms of the Fabry and Buisson standards used for H and K, it was possible to obtain the factor for reducing Adams' wave-lengths to the same scale.

The values for H and K found by Adams from the last eight plates were $\lambda 3968.628$ and $\lambda 3933.817$, respectively, which become $\lambda 3968.477$ and $\lambda 3933.668$ on the absolute scale. These differ from the results given in this paper for H and K by 0.001 Å only. Mr. Adams used a different combination of portions of the same apparatus to form the Littrow spectrograph with which his measurements were made.

CALCIUM IN THE SPARK

The same standards were used in the determination of the wave-lengths of the H and K lines in the spark as in the arc. The spark was produced between terminals of metallic calcium for the absorption lines, and between copper terminals moistened with a dilute solution of calcium chloride for the bright lines, which were purposely made of weak intensity. The source was a high-voltage transformer charging an oil-immersed condenser. In series with the spark there was a

TABLE II

Plate	H	λ 3948	K
4..............	3968.476 r	3948.783	3933.668 r
5 (1)............	.474 r	.781	.668 r
(2)............	.476 r	....	.672 r
6 (1)............	.475 b	.783	.668 b
(2)............	.476 b	.784	.666 b
7 (2)............	.474 r	.781	.665 r
(3)............	.470 r	....	.672 r
Means...........	3968.476	3948.782	3933.668

variable self-induction. The comparison spectra were from an iron arc. A lens kept in a fixed position formed the image of the spark and the arc on the slit, a mirror being introduced into the optical train to bring the light from the arc into the axis of the system. The spectrograph was the 13-foot Littrow instrument in the Pasadena laboratory, referred to as spectrograph No. 1. The plates were measured on the same machine and in the same manner as for the calcium arc plates. Table II exhibits the results.

The agreement between these results and those for the calcium arc

is extremely good, particularly in view of the character of the spark lines when compared with the arc lines, and the lower dispersion used. It indicates the accuracy of a grating for comparative measurements over a limited region when such excellent standards are available, and shows the practically identical wave-lengths of H and K in the arc and spark. The control line, λ 3948 of iron, serves in this case only as a check on the measurements of the comparison spectrum and the reduction of the plates. That there was no instrumental shift is shown by the agreement of the plates with each other.

CALCIUM IN THE ELECTRIC FURNACE

Dr. King placed at the writer's disposal a plate of the region of the spectrum under investigation taken with calcium in the electric furnace at an approximate temperature of 2600° C. The graphite tube was a new, clean one in which some iron dust had been placed that furnished a comparison spectrum showing four of the standards previously used, viz., $\lambda\lambda$ 4021, 3977, 3906, and 3865. The weakest line of the five, λ 3935, was absent. The line λ 3930, whose wave-length was determined in the first part of the investigation in terms of the Fabry and Buisson standards, was used in place of λ 3935, so that the results should be strictly comparable with those for the arc and the spark. The H and K lines were from impurities in the graphite. On this plate the lines of calcium were bright. The results of the measurements are given in Table III.

TABLE III

Exposure	H	λ 3948	K
1	3968.479	3948.789	3933.667
2	.470	.772	.661
3	.472	.787	.667
4	.473	.775	.663
Means	3968.474	3948.781	3933.665

The plate was not taken with a view of wave-length measurements and was not as satisfactory as plates taken expressly for the purpose would have been. In every exposure one or more of the standards was very difficult to set on and the accuracy is much less than that of the corresponding arc measurements. Nevertheless, the final result

shows a satisfactory agreement with the arc measurements, considering the quality of the plate.

In Table IV are assembled the results from the arc, spark, and furnace measurements. The figures in parentheses indicate the number of lines on which the measurements were based.

TABLE IV

Source	H		K	
	Absorption	Emission	Absorption	Emission
Arc..........	3968.476 (23)	3968.474 (2)	3933.667 (23)	3933.667 (2)
Spark........	.476 (5)	.476 (2)	.669 (5)	.667 (2)
Furnace......	....	.474 (4)	.665 (4)	.665 (4)
Means........	3968.476	3968.475	3933.667	3933.666

In determining the following means equal weights were given the results from arc, spark, and furnace, respectively. The final weighted means are:

$$H=3968.476 \qquad K=3933.667$$

The close agreement of the results for the three sources, the two classes of lines and the three spectrographs, shows that the wavelengths of the H and K lines of calcium are independent of the source of the radiation and the density of the vapor, and that gratings of high dispersion may be depended upon to give results over a limited region with an error not greater than 0.001 Å when the best lines referred to sufficiently homogeneous standards.

CHARACTERISTICS OF THE H AND K LINES

The appearance and behavior of the H and K lines of calcium are very similar under all conditions; both, as King has shown,[1] are high-temperature lines, and in the furnace increase in strength in proportion to the temperature. They shift equally under pressure, but only to one-half the amount that λ 4227 shifts for the same increment of pressure, as Humphreys has shown.[2] But in the magnetic field H consists of four and K of six components.[3] Both lines are charac-

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 32, p. 7; *Astrophysical Journal*, **28**, 389, 1908.

[2] *Astrophysical Journal*, **3**, 114, 1896.

[3] Kayser, *Handbuch der Spektroscopie*, **2**, 671.

teristic of the rarer vapor of calcium, as shown by Sir William and Lady Huggins, who found that the other calcium lines disappeared, leaving H and K alone, when by repeated washings they reduced the quantity of calcium vapor in the spark between platinum poles moistened with a dilute solution of calcium chloride. They state:

> Once more the electrodes were washed, with the expectation of having removed completely the last of any trace of calcium. To our surprise when the photograph was developed the H and K lines came out alone.[1]

The extraordinary persistence of these lines was shown in the present investigation when using the arc between copper poles moistened with a drop of very dilute solution of calcium chloride. After repeated washings the lines still appeared on the plates and the observations of Liveing and Dewar, as given in Professor Liveing's letter to Sir William Huggins,[2] could not be confirmed. In this he says:

> I have been looking up some observations of Dewar's and mine on the H and K lines made in 1879. We found that when we used for the arc carbon poles which had been heated for two days in chlorine to remove metals the calcium lines were not at first visible in the arc, but after a time H was seen alone and not strong; after a further time K was seen and then other calcium lines came out. No doubt the calcium had been pretty well removed from the carbon rod to some depth, but not from the interior, so that as the carbon burnt away in the arc the calcium in the interior became manifest.

They found a similar behavior for the H and K lines on diluting the calcium vapor in the arc by a stream of hydrogen, the H line remaining longer as the proportion of hydrogen increased, and appearing before the K line as the proportion of hydrogen was decreased. In the present investigation it has not been possible to obtain either of the lines alone on the photographic plate. K is always wider and stronger than H, even when the lines are extremely narrow and faint. From the measurement of twenty-five emission and absorption lines, the mean ratio of the width of K to H is 1.28. A wide range of widths was covered, which, expressed in Ångström units, are as follows:

[1] *Atlas of Representative Stellar Spectra*, p. 98.

[2] *Loc. cit.*

	Emission		Absorption	
	K	H	K	H
Maximum width	0.455	0.405	0.143	0.106
Minimum width	0.072	0.065	0.056	0.040
Mean width	0.177	0.147	0.087	0.064

The mean ratio of the widths for absorption lines is 1.36, and for emission lines 1.20. The difficulties of width measurement are great and are different for the two classes of lines, and this difference in ratio was at first attributed to errors of measurement, but a similar result was found for the ratio of the intensities of the emission lines. The intensity ratio of bright K to H was obtained by comparing them directly with an artificial scale, made, on the suggestion of Mr. Adams, by using a narrow slit and photographing the direct reflection of sky light from the grating, with exposure times varying from one to sixty seconds. For eight lines of mean width 0.160 Å, the ratio of the intensity of K to H was 1.23; for six lines of mean width 0.083 Å, the ratio was 1.70. In the case of emission lines, K is always stronger and broader than H, and with extremely small quantities of vapor in the arc the preponderance of K over H increases, both in width and intensity. The narrow absorption line produced by the outer cool and thin layer of the arc corresponds to the emission line with rare vapor, and shows a similar preponderance of K over H in width and intensity.

The reversal of K is always stronger than H, and often shows a clean reversal when H shows scarcely a trace of reversal; and in the case of a double reversal, as described later, the emission line of K showing in the reversal is much stronger than the corresponding line of H. In view of the greater width and intensity of K for minute quantities of calcium vapor in the arc, it is difficult to understand how K could disappear before H, as was observed by Liveing and Dewar, unless the observations were made visually The wording in Professor Liveing's letter indicates visual observations, as he speaks of their being "visible" and "seen" In that case it might easily happen that the line of shorter wave-length would disappear first, as the sensibility of the eye falls off very rapidly in the region of H and K.

DOUBLE REVERSALS

Humphreys obtained double reversals of the H and K lines of calcium by using two arcs in series, the one nearer the slit containing a small amount of calcium while that 5 cm farther away contained a large quantity.[1] Barnes attempted to obtain double reversals of these lines by the method of Konen and Hagenbach,[2] which consists in taking a plate of very short exposure of the arc when quickly remade after having been extinguished and just on the point of burning, but he was unsuccessful.[3] In the course of the present study four cases of double reversal of the H and K lines were observed in photographing the spark. The spark was between terminals of metallic calcium and was projected on the slit by a lens so that the image was slightly larger than the spark. The terminals were 5 mm in diameter and pointed, and the gap 5 mm long. The spark was explosive and very brilliant, except occasionally for a few seconds when it was in a hissing state and much less vapor was being evolved It is very doubtful whether the conditions for a true double reversal were present. It seems more probable that the absorption lines were produced by the outer and cooler layers of vapor during the explosive condition, and the fine sharp emission lines were superposed during the hissing stage—though this last was of short duration compared to the total exposure time of about sixty seconds—or possibly during the occasional displacements of the image when for an instant the edge would be on the slit. Out of hundreds of plates taken with the arc, only one showed a double reversal, and that was probaby caused by an extinction and remaking of the arc, as in the Konen and Hagenbach method.

ABSOLUTE WAVE-LENGTHS OF SOME IRON LINES

During the investigation the wave-lengths of nine lines of iron in the region near H and K were carefully determined from the five Fabry and Buisson standards and may serve as additional reference lines in this region, as they are well adapted for measurement. In the following list the secondary standards of Fabry and Buisson are

[1] *Astrophysical Journal*, **18**, 204, 1903.

[2] *Ibid.*, **19**, 111, 1904.

[3] *Ibid.*, **27**, 156, 1907.

marked with an asterisk. The figures in parentheses indicate the number of plates on which the corresponding lines were measured, and the letter "r" the lines which are easily reversed.

4021.872*	3940.882 (6)
3977.745*	3935.818*
3969.261 (10) r	3930.301 (25) r
3966.068 (10)	3927.921 (10) r
3956.678 (4)	3922.913 (9) r
3956.462 (10)	3906.481*
3948.783 (25)	3865.526*

Mr. Kristian Lows was a visitor at the observatory for some weeks during the past summer. The writer wishes to express his appreciation of the assistance rendered by Mr. Lows in making some of the observations and in the reduction of plates for this investigation.

SUMMARY OF RESULTS

1. The wave-lengths of the H and K lines of calcium are 3968.476 and 3933.667, as determined from the secondary standards of Fabry and Buisson, with an uncertainty of not more than 0.001 Å.

2. The wave-lengths are identical for absorption and fine emission lines, and are the same for the arc, spark, and furnace; that is, they are independent of the density and temperature of the vapor, and the source of the radiation.

3. The mean ratio of the width of K to H is 1.28, and the mean ratio of the intensity of K to H 1.47, for the lines measured. Both H and K persist in the arc as the density is decreased, with an increasing preponderance of K over H in width and intensity, pointing to the disappearance of H before K.

4. With sufficiently homogeneous standards, an accuracy of 0.001 Å can be obtained with a grating of high dispersion over a limited region of the spectrum, in the case of the best lines.

Mount Wilson Solar Observatory
December 1909

SOME PHOTOGRAPHIC PHENOMENA BEARING UPON DISPERSION OF LIGHT IN SPACE

By HERBERT E. IVES

Star photographs obtained by Tikhoff have been interpreted as evidence for the dispersion or scattering of light in space.[1] Photographs of the same constellation were made through differently colored glasses. In a "red" photograph the faint stars were more numerous than in a "blue." Therefore, since the faint stars are on the whole more distant, and since blue light is scattered more by passage through a turbid medium than is red, this could mean that an appreciable scattering of light takes place in space.

Considerable discussion in astronomical and physical journals has been aroused by this work. Of most interest from the standpoint of the present article is the discussion by J. A. Parkhurst[2] of the photographic side of the question. The exponent p of the Schwarzschild equation $It^p = iT^p$, upon the different values of which Tikhoff founded part of his conclusion, is shown by Parkhurst to have a wide range of values for different densities upon the same plate, values as large as Tikhoff's "red" values, and as small as his "blue." Photographs by Parkhurst of the *Pleiades* (the constellation chiefly cited by Tikhoff) through visual color-filters and without filter, magnitudes being measured on an absolute scale, showed no evidence for space dispersion. The conclusion he drew was that "the cause lies somewhere in the instruments and plates used, with a probability that it is mainly in the plate and filter." The exact cause was, however, not determined. It is the object of the present paper to show how results may be obtained similar to those of Tikhoff, due to peculiarities of the photographic plate.

In Tikhoff's work the assumption is made that the scale of gradation of the photographic plate is the same for all colors. In other words, the relative photographic action by two differently colored light-sources will be the same, no matter what the time of exposure or

[1] *Comptes Rendus*, **148**, 267, 1909.

[2] *Astrophysical Journal*, **30**, 33, 1909.

the absolute intensity. According to Eder and to Abney[1] this is not so. Abney thus expresses the conclusions from his experimental work: "The least gradation was given at the wave-lengths to which the salt was most sensitive." Leimbach,[2] in the most recent investigation in which this matter is treated, finds no difference for different colors. The evidence is, therefore, apparently conflicting. There is, however, the possibility of these different results being explainable as due to differences in the manipulation of plates. If so, the manipulation in the astronomical work in point may have been such as to cause differences of gradation, as noted by Eder and by Abney.

Two photographic phenomena with which the writer has been familiar presented themselves as possible causes of a different scale of gradation for different colors. First is the fact that with certain developers and plates an image obtained through a red glass develops more slowly than one through a blue glass, although the final density is the same. This is sometimes referred to as a photographic "Purkinje effect." Second is the fact that, in a plate sensitized by bathing, the sensitive layer is very thin. This is shown by microsections obtained by the writer in studying the Lippmann film.[3]

An explanation of the first effect which offers itself is that the effective thickness of film is different for the two colors. The bromide of silver emulsion is comparatively opaque to blue light, transparent to red. We might, therefore, expect the photographic action of blue light to become progressively less through the film, while that of red light is more nearly constant. By long development the red image would continue to gain density by the penetration of the developer into the film after the whole depth of the blue image had been reached. (The red sensitizer is assumed to be incorporated in the emulsion.) There would result upon development to the same densities for the most exposed parts a red image of greater depth than the blue image. A thick layer of emulsion having a longer scale of gradation than a thin one, we should expect a longer scale for the red than for the blue.

If this hypothesis is borne out by experiment we would have an effect just the opposite of that obtained by Tikhoff. His photographs

[1] *Instruction in Photography*, p. 452.

[2] *Zeitschrift für wissenschaftliche Photographie*, **7**, 205, 1909.

[3] *Astrophysical Journal*, **27**, 348, 1908.

would therefore indicate a larger dispersion than he deduced. If, however, we investigate the bathed plate with the same idea of film thickness in mind we find a different condition. The layer sensitized by bathing being thin, it might well happen that the emulsion layer sensitive to red was thinner than that sensitive to blue. This would give a shorter scale to red than to blue.

With this theory as a guide the experiments described below were carried out. Since for the effects to be marked the developer should act through the depth of the film, slow, dilute developers of the non-fogging type were employed, hydrochinone at first, and later glycin (as giving less fog). Development lasted from twenty to thirty minutes. In certain cases, to test the theory, other methods of development were tried, as noted in the appropriate places.

To investigate the scale of gradation, simultaneous exposures of increasing length were made, through red and blue glasses, upon the same plate. A long, narrow plate-holder ($2\frac{1}{2} \times 8$ inch plate), sliding in a wooden frame, permitted successive portions of the sensitive plate to be exposed by means of a "Volute" shutter to the light of a 16-candle-power incandescent lamp eight feet distant. The lamp was maintained at constant voltage throughout a series of exposures. The blue glass was a combination of cobalt blue and signal green glass; the red a methyl-violet and tartrazine stained gelatine film on glass. Neutral-tint glasses placed over either red or blue made it possible to obtain exposures of any relative magnitude desired. Exposures in a series ranged from $\frac{1}{2}$ to 60 seconds. Densities were measured in a spectrophotometer in which the comparison light was varied by a Brodhun sector. The opacities plotted are the total opacities of glass, gelatine, and reduced silver.

The results of the investigation are given diagrammatically in Figs. 1 and 2. With a Cramer "Instantaneous Isochromatic" plate, developed as described, the red gradation was steeper than the blue. With a "Seed 26" bathed for two minutes in a $\frac{1}{100{,}000}$ solution of pinacyanol the blue gradation was steeper than the red. In both the characteristic curves given the relative exposures were so adjusted that the curves crossed. By proper adjustment the red and blue densities could be made equal for the longest exposures, in which case, with the bathed plate, the less exposed blue images

were very much fainter than the corresponding red; with the isochromatic plate the reverse condition held. These effects are also obtained if instead of varying exposure the intensities are varied, as was tested by exposing plates at different distances from the lamp, the time of exposure remaining constant.

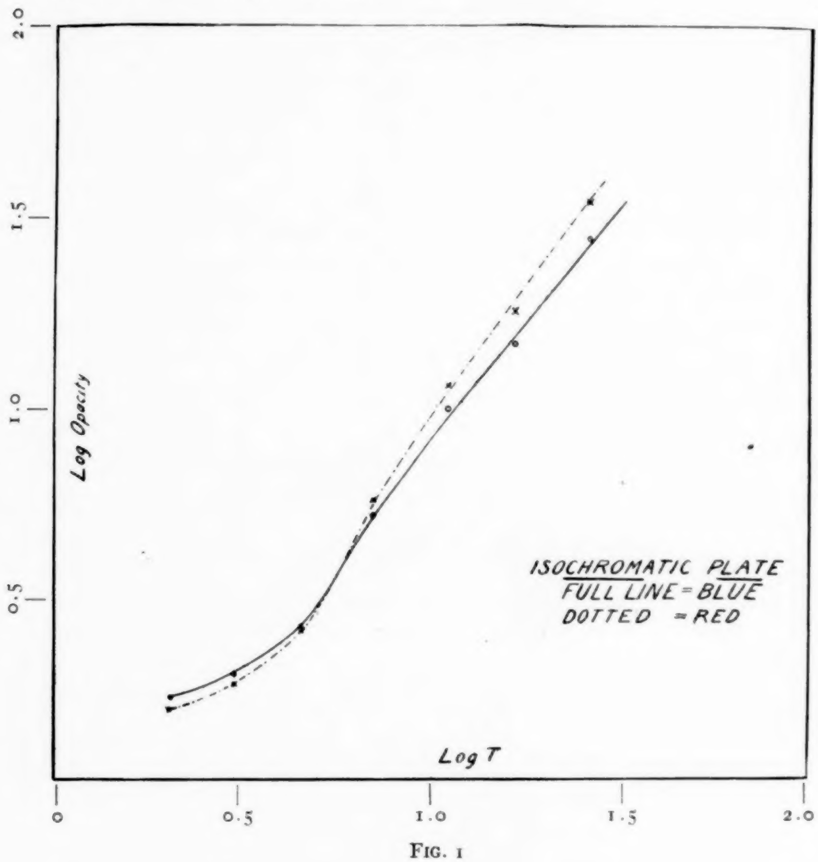


FIG. 1

The conclusion to be drawn from these experiments is this: The relative photographic action of different colors is different for different exposures and for different absolute intensities. With an isochromatic plate we may have a "Purkinje effect," with a bathed plate we may have the reverse of the "Purkinje effect." In the existence of these effects we have a possible explanation of the photographs

obtained by Tikhoff. At any rate, their existence shows the necessity for ascertaining the properties of the plates which he used, under the conditions which held, before accepting conclusions from his results. His plates were bathed, and his results were of just the kind obtained in this investigation with bathed plates.

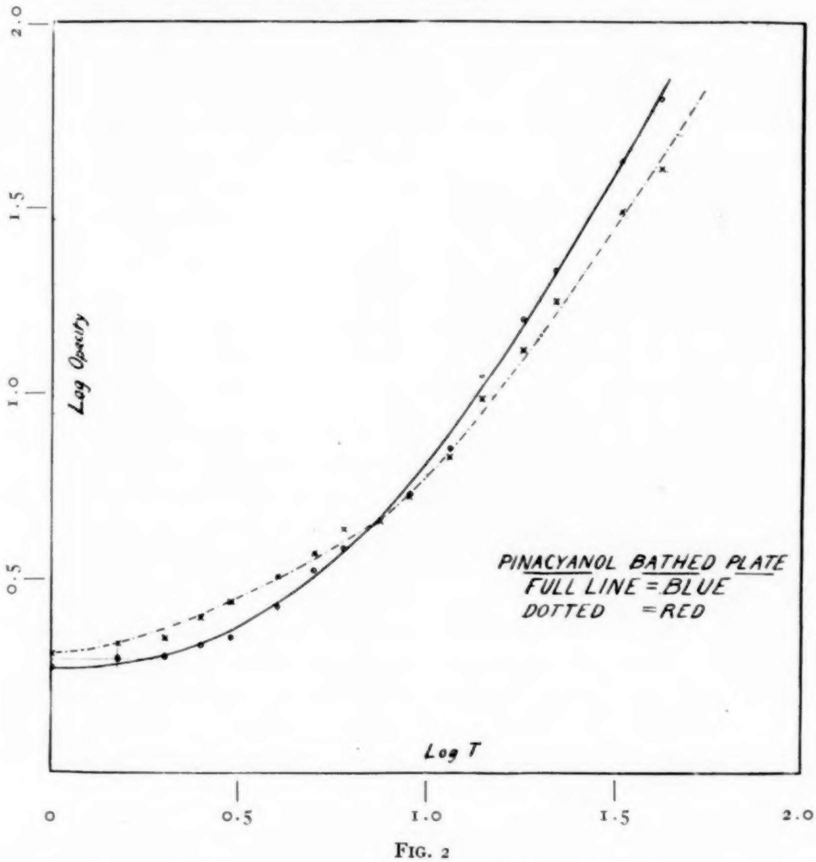


FIG. 2

With a view to testing the validity of the theory upon which the experiments above were based, several others were performed. An isochromatic plate was developed with pyrogallic acid, which, as shown by the writer's sections of Lippmann films, acts chiefly on the surface of the film up to the point of fogging. It was found necessary greatly to increase the red exposure to secure the same density in red

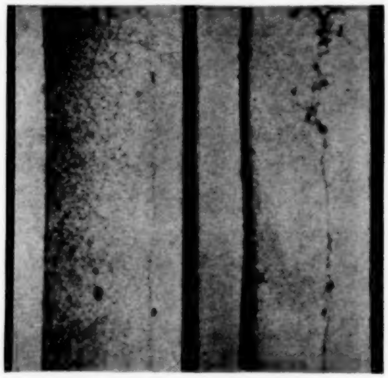
as in blue. When this was done the difference in gradation of the two was no longer evident. In each, therefore, the effective film depth was the same. Another plate was developed with strong hydrochinone, but the development limited to two minutes, with a similar result. These experiments, while appearing to confirm the theory worked upon, also offer an explanation of the conflicting results of Abney and Leimbach. The latter used quite short development with ferrous oxalate in order to avoid all trace of fog. Longer development, or perhaps experiments with other developers would have given other results.

A result obtained with another plate, a slow "Cramer Spectrum," does not appear to fit in so well with the film-thickness theory. These plates are exceedingly sensitive to red, with the sensitizer in the emulsion. With them no difference in scale of gradation was appreciable with any development. With them, however, it is not necessary to depend upon the whole thickness of film to secure sufficient density in the red to match the blue. For the same developed density it is probable that the effective film thickness was nearly the same. Of course it is possible that other factors than film thickness enter in certain cases. There may be a "specific gradation" of the sensitive emulsion for each wave-length, more evident with some sensitizers than with others. If this is the case, Abney's statement of the manner in which gradation changes with color may be nearer expressing the fact than the explanation in terms of thickness. In any case the thickness is of significance. If the thin layer of sensitiveness in the bathed plate is to give as much photographic action as the thicker layer of the unbathed plate, the sensitiveness per unit volume must be greater. We have then actually the condition stated by Abney, that the least gradation is given for the color to which the salt is most sensitive. It is also possible that in the "spectrum" plate, either the sensitizers or the method of manufacture may tan the surface, as pyrogallic acid developer does, and so retard the penetration of the developer. A different amount of photographic inertia for different colors may also play some part.

Whatever the complete explanation of the effects obtained, it is made evident by microscopic sections of the films that the different relative thickness of bathed and unbathed emulsion is a reality, and

therefore probably responsible for a large part of the effects. In Fig. 3 are given two microsections of film from the same plate, *a* for blue light, *b* for red. The small depth of the bathed sensitization is evident. In an isochromatic emulsion the film shows a section very similar in red and blue.

The question naturally arises: What is the magnitude of these two effects? Are they sufficient completely to account for the results obtained by Tikhoff? The extent of the difference in photographic action for different colors obtained by him is unfortunately given only by the values derived for the Schwarzschild exponent p. This, as Parkhurst has shown, is indefinite; widely different values may be obtained where no color difference enters. The writer, on measuring several plates exposed in a manner to give data for calculating p, found the same conditions noted by Parkhurst; a different value could be obtained from each density represented. For the bathed plate p was larger for red than for blue, but the values were so unsatisfactory as values of a supposed constant that no weight could be attached to them. It is indeed open to question whether this exponent, which is a constant for the region of normal exposure, is necessarily so for the region of comparative under-exposure, in which it is evident from the characteristic curves lie the photographic effects with which we are concerned. Another basis of comparison was, therefore, adopted. A bathed plate was exposed through red and blue glasses for the same length of time at varying distances from a source of light, thus securing a gradation scale due to different intensities. The density values were plotted against intensities, giving crossing curves similar to those of Fig. 2. From the curves it was possible to determine the relative intensities for the two colored lights which would give any chosen density. The curves were then replotted with the intensity values for one color multiplied



a *b*

FIG. 3

by a constant factor, so that the curves crossed at a point of considerable density. This is exactly equivalent to making the exposures alike for high intensity by the interposition of the proper neutral tint glasses. With the intensity corresponding to this density (about 3 per cent. transmission) as unity, the intensities corresponding to successive star magnitudes (each 0.4 of the one above) were marked off. The curves then showed that the decrease in density corresponding to a drop of five magnitudes in the red was given by a drop of just over four magnitudes in the blue. Therefore, if two such plates, exposed through red and blue glasses for such times as would give the same density for first-magnitude stars, were examined for the apparent magnitudes of fainter stars, a difference of a whole magnitude would be found by the time the fifth magnitude was reached. This is a difference which should be plainly evident on inspecting the negatives, as Tikhoff says was the case with his photographs.

The conclusion from this work is that the assumption underlying Tikhoff's experiment—that the scale of gradation of the photographic plate is the same for all colors—is not true. The relative photographic action of different colors depends upon the time of exposure and the absolute intensity. The experiment performed by Tikhoff has meaning only if the scale of gradation of the plates employed is known for each color, under the conditions used, and allowance made for it. This is equivalent to the use of such an absolute system of measuring photographic magnitudes as that of Parkhurst and Jordan,[1] provided, of course, that the comparison densities are obtained for each color used. The "Cramer Spectrum" plate, which is highly red sensitive, though not bathed, is apparently largely free from these spurious effects, and would be much better adapted for a research of this kind than an isochromatic of low red sensitiveness, or a bathed plate. In every case the plates employed should be subjected to careful laboratory test before using.

It is evident that a difference in the relative photographic densities of faint and bright stars by differently colored light may be entirely a photographic phenomenon, and hence no evidence for scattering of light in space. Should, however, the effect be found to be real, when tested under conditions as indicated above, there is another possible

[1] *Astrophysical Journal*, **26**, 299, 1907.

explanation. According to Tikhoff's reasoning the faint stars are on the whole more distant. Now we know that many faint stars are as near as some of the brighter ones; certainly in many stellar groups faint and bright stars are grouped together at about the same distance from us. Could we not then state with equal justification that the faint stars are on the whole the smaller ones? Being smaller they would in any group of common origin be farther along in their life-history and so, it might be argued, cooler, and more yellow. The same photographic test which has been applied to the question of light scattering might, therefore, equally well have been called upon to test the hypothesis that the faint stars are as a class smaller than the bright ones—had such a hypothesis been necessary to astronomical problems. If this reasoning is correct, positive photographic evidence of the kind we have been considering would not alone be sufficient to prove scattering of light in space. Reliable conclusions could be drawn only by knowledge of the size and distance, as well as the color, of a large number of stars, the line of investigation which Kapteyn[1] is now pursuing.

PHYSICAL LABORATORY
NATIONAL ELECTRIC LAMP ASSOCIATION
CLEVELAND, OHIO

[1] *Astrophysical Journal*, **30**, 284, 1909.

ON THE APPLICATION OF THE LAWS OF REFRACTION IN INTERPRETING SOLAR PHENOMENA

By J. A. ANDERSON

During the past few years Professor W. H. Julius has published a number of very interesting papers dealing with the subject of refraction and anomalous dispersion in the atmospheres of the sun and other heavenly bodies (*Astrophysical Journal*, from Vol. **12** onward). The conclusions at which he arrives are in many cases radically different from the views which have hitherto been held by astrophysicists, and it is not surprising therefore that the investigators in this subject have been rather slow in adopting them. They are naturally unwilling to discard their own views in favor of new ones until these new ones shall have been established beyond any possibility of doubt. Now, the earlier experiments of Julius, although they illustrated the principles of anomalous dispersion admirably, did not imitate the conditions existing in the atmosphere of the sun well, because the source of white light invariably subtended only a very small angle as seen from the position occupied by the refractive medium in question, while in the supposedly parallel case in the solar atmosphere the angle subtended is very nearly 180°. In a later paper by Julius, however, entitled "Regular Consequences of Irregular Refraction in the Sun,"[1] a source of much greater angular diameter is used, although the description of the apparatus does not enable one to say just how closely conditions existing in the solar atmosphere are copied.

In the present paper we shall make an attempt to examine by elementary methods the consequences of irregular refraction in such an atmosphere as that of the sun, and then try to define the conditions which an experiment must fulfil if it is to have any application to solar phenomena in general.

Let us assume in the first case that what is called the photosphere may be represented by a perfectly uniform self-luminous surface. Any point in the solar atmosphere (neglecting the corona) lies so

[1] *Proc. of Roy. Acad. Sci. of Amsterdam*, **18**, 266, September 25, 1909.

close to the photosphere that the latter subtends an angle very nearly equal to 180° and we may therefore fairly represent conditions by replacing the photosphere by an infinite, plane, self-luminous surface. The irregularities in the solar atmosphere may be assumed to be of the nature of "Schlieren" which are usually more or less nearly circular cylinders of gas whose refractive index differs from that of the surrounding medium, the change in refractivity being gradual from the circumference toward the axis.

Let A (Fig. 1) represent a section of such a cylinder, and $BCDE$ represent the path of a ray of light passing through it. As far as the

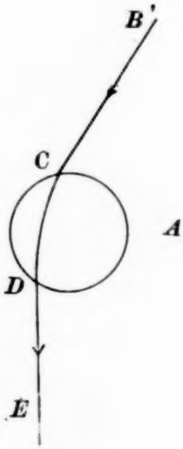


Fig. 1

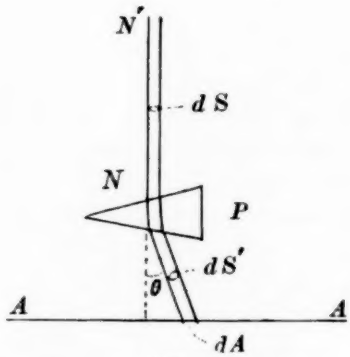


Fig. 2

ray $BCDE$ is concerned the action is the same as that of a prism of suitable angle, and for the sake of clearness we will first suppose this cylinder replaced by a prism. After finding what is the action of a prism, we will make the changes necessary to fit the case under consideration. Let AA (Fig. 2) be the uniform plane self-luminous surface, and let P be a prism of any transparent substance whatever, so oriented that light of wave-length λ emerging from it in the direction NN', which is perpendicular to AA, shall have passed through it with minimum deviation. Consider a small cylinder of the emerging beam of cross-section dS, trace this back through the prism to the surface AA, where it incloses an area dA. Call the cross-section of

the cylinder between P and AA dS'. Since the beam has traversed the prism with minimum deviation we have $dS=dS'$. If the condition for minimum deviation had not been fulfilled we should have dS greater than dS' or dS less than dS' according as the angle of incidence is greater or less than the angle of emergence. For the case of minimum deviation we have, letting θ be the angle of deviation, $dS'=dA \cos \theta$. Let I_λ be the intensity of light of wave-length λ emitted in a direction normal to AA, then the intensity in a direction making the angle θ with the normal will be $I_\lambda \cos \theta$, and hence the amount of energy passing through any section of our cylinder between AA and the prism per second will be $dA \times I_\lambda \cos \theta = dS' \times I_\lambda$. If we now neglect losses by reflection from the surfaces of the prism we have for the energy passing any section of the cylinder NN' per second $I_\lambda dS$, since $dS=dS'$; that is, the intensity is I_λ or the same as it would be in the absence of the prism.

If the light did not traverse the prism with minimum deviation we should have the intensity as seen through it greater or less than what it would be in its absence according to whether the angle of incidence is less or greater than the angle of emergence. For small deviations, however, the change in intensity would be slight, a deviation of 10° producing a change in the intensity which can never exceed $1\frac{1}{2}$ per cent.

For an actual prism the above statements will require some slight modification owing to the reflections at the two surfaces, but if we substitute for the prism a mass of gas such as that represented in Fig. 1, then, since the change in refractive index is gradual, we have no reflection and the above statements are correct. Moreover, for the case represented in Fig. 1, no ray of light can pass through except at minimum deviation, and this is very approximately true even if the cross-section instead of being circular is distorted into the form of an ellipse. Therefore it is evident that *a uniform luminous surface would appear uniformly luminous to an eye at a distance, even if covered by an atmosphere full of "Schlieren," provided the deviation produced by the "Schlieren" does not exceed 90°.*

Since this holds for light of any wave-length, it will hold for such substances as the metallic vapors showing anomalous dispersion, except inside the limits of the absorption band. In this region absorp-

tion would of course take place, extinguishing the light more or less completely. But ***unless the deviation for light anomalously refracted exceeds 90°, it follows that the absorption band would have exactly the same width and character that it would have if produced by a perfectly homogeneous atmosphere of the same absorptive power.***

This applies directly to the center of the solar disk. It also applies to any point of the sun's visible surface if we substitute for 90° the angle at the center of the sun between the radii drawn to the point in question and to the nearest point on the limb, respectively. ***Now, since such phenomena as sun-spots, faculae, flocculi, etc., do not change materially in appearance as they approach the limb it becomes at once evident that the angles of deviation with which we have to deal never approach 90° and possibly never even 5° or 10°.***

We can get a better idea of the order of magnitude of the deviations to be expected by inquiring what irregular density-gradients we may reasonably expect to find in the solar atmosphere. In the paper by Julius referred to above it is shown that if we have a density-gradient equal to the vertical gradient at the earth's surface, the radius of curvature of a ray at right angles to the gradient will be about $\frac{1}{50}$th of the radius of the photosphere. There are two causes which tend to destroy irregular density-gradients in an atmosphere. One is the pressure of the gas, and its effect will be the same on the sun as on the earth, since it depends upon the inertia of the gas. The other one is the weight of the gas, which causes it to move toward its own proper level in the atmosphere. This is 27.3 times as great on the sun as on the earth, and, besides, the distance the gas will have to move to find its own proper level is only $\frac{1}{27.3}$ as great as on earth. It follows therefore that irregular density-gradients in the sun's atmosphere will have, in general, only $\frac{1}{(27.3)^2}$, or about $\frac{1}{750}$ of their value in the earth's atmosphere. These gradients in the earth's atmosphere perhaps never exceed one inch of the barometer in ten miles (1 cm in 6 km), or about $\frac{1}{50}$th of the vertical gradient at the surface. The corresponding radius of curvature for a light-ray at right angles to the gradient would equal the radius of the sun's photosphere. For the gradients to be expected in the sun's atmosphere the radius of curvature would be 750 times this, or about 325 million miles (525 million kilometers).

Let us allow a ray of light a path of 10,000 miles (16,000 km) in a direction at right angles to this gradient, *and the corresponding deviation amounts to just about 6 seconds of arc!* In cases of light-rays suffering anomalous dispersion the deviation may perhaps reach 100 times this value, or a matter of a few minutes of arc. And this on the extravagant assumption of an undisturbed path 10,000 miles long!

In conclusion we may say, then, that irregular refraction and anomalous dispersion undoubtedly do modify the true appearance of the solar surface, but the order of magnitude of the effect is such that with our present instrumental equipment it is very doubtful if we shall be able to detect it.

Physical Laboratory
Johns Hopkins University
January 18, 1910

GLASS AND METALLIC REPLICAS OF GRATINGS

By J. A. ANDERSON

Replicas of gratings were first made by Thorpe in England; later by Wallace and by Ives in this country. The method used by both Wallace and Ives is to pour upon the grating a solution of gun cotton in amyl acetate, or some similar substance, and after this is dry to allow it to peel off under water, and then to mount it upon a piece of plane glass. One surface of the film of collodion, the one which was in immediate contact with the surface of the grating, is found to be a fairly accurate copy of the ruled surface of the grating itself, while the other one is more or less perfectly flat. The first will be spoken of simply as the ruled surface, or as the face.

Thorpe mounted his replicas with the ruled surface up, while Wallace speaks of mounting the film either ruled surface up or down, preference being given to the latter. Ives mounts all of his with the ruled surface down, in contact with the glass, I believe.

When a replica is mounted face up it may be transformed into a metallic reflection grating simply by coating it with platinum by means of cathode disintegration in a vacuum, as has been lately described by E. Gehrcke and C. Leithäuser.[1] As a rule, however, this surface is perhaps never quite plane, owing to the unavoidable differences in the thickness of the film in different places, and hence I imagine that gratings made in this way will never perform very well when subjected to a really severe test.

The method of making the ordinary transmission replicas as used by Wallace and Ives has already been described by Wallace[2] and hence need not be described here in any more detail.

During the past two years the author has been experimenting with the making of replicas in the hope of finding a method of duplicating the gratings in metal; and in the course of the work a method was also found by which gratings can be copied in glass or quartz, which of course makes them very permanent. Besides this, much

[1] *Verhandlungen der Deutschen Physikalischen Gesellschaft*, 1909.

[2] *Astrophysical Journal*, **22**, 123, 1905; **23**, 96, 1906.

was learned about the characteristics of gratings and their replicas which must have escaped other workers in the same field, since the author had at his disposal about 100 Rowland gratings to work with, while others have had only a very limited number. This will, however, be the subject of a future paper. At present we shall touch on only one point which is important in connection with the subject of this paper, and that is, "How perfect is it possible to make a replica?" or, what amounts to the same thing, "How nearly will the resolving power of a replica equal that of the grating from which it was made?"

At first some small gratings of $1\frac{1}{4}$ inches and $2\frac{1}{2}$ inches width were used and it was found that the replicas if carefully handled were all good when examined by an ordinary laboratory spectroscope having about a 1-inch objective of perhaps 10 inches focal length. On trying them with our plane grating spectroscope whose focal length is about 10 feet it was soon seen that they fell far short of equaling the original grating in definition and resolving power. This became more and more evident as larger gratings were used. The replicas from a 6-inch flat grating were practically worthless, although if an inch square or so of their surface was used it performed beautifully. The explanation is evident. The films in drying shrink somewhat and tend to shrink a little unevenly, so that the lines are no longer absolutely straight, parallel, and equidistant, but deviate from these conditions more or less. This phenomenon can be made very evident to the eye by placing the replica in contact with the original grating so that the lines of the two are parallel to each other. In this case a series of dark and bright fringes are seen where the lines of the replica alternately get in step and out of step with the lines of the grating. Wallace mentions that in his replicas these fringes are always more or less curved but that if the curvature is small the replica is found to perform well.

The author attempted to make these fringes all parallel and equidistant by mechanically stretching the replica wherever it was required, this being of course done before all the water between it and the glass surface had evaporated. At first he succeeded only in making things worse, but with a little practice it was soon found that a replica could be corrected to within a small fraction of a fringe,

which of course corresponds to the same small fraction of the grating-space in the replica itself. A replica corrected in this manner gives the same resolving power as the grating from which it was made within a very few per cent., provided that the glass on which it is mounted is optically perfect.

In order to transform one of these replicas into a metallic grating it is not sufficient simply to turn its face up and then deposit a film of metal over the ruled surface, since the film is never of absolutely even thickness and hence the ruled surface in this case will not be plane, as it must be, accurately, if the resulting grating is to be of any value; nor is it sufficient to silver the glass surface upon which the replica is placed face down, for very obvious reasons. What is wanted is some substance which will fill up the grooves *AA* (Fig. 1) between the replica and the glass surface after the replica has been corrected and dried, and which will adhere to the glass sufficiently

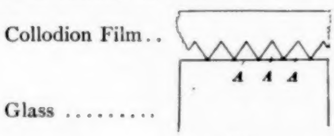


FIG. 1


FIG. 2

to allow the replica to be stripped off. The result will then be similar to Fig. 2. This is evidently a fair copy of the grating from which the replica was made.

Such a substance was accidentally found. If certain gums are dissolved in the collodion solution they will, on gently heating the glass plate upon which the corrected replica is placed, slowly ooze out, filling up the grooves as indicated, and on cooling will harden and allow the replica to be stripped off. The resulting grating (Fig. 2) may now be treated in one of the two following ways:

1. It may be covered by a thin film of platinum, nickel, or other suitable metal in a vacuum, which coating may be improved by subsequent electroplating, thus producing a durable metallic grating having a perfect optical surface.

2. It may be treated with hydrofluoric acid gas, thus transforming it into a glass or quartz transmission grating with an equally good optical surface, as the gum used is not affected by the acid, while the glass or quartz between the ridges is rapidly attacked.

In conclusion it may be remarked that the method is equally well applicable to concave gratings, the apparatus required being a convex surface of the same radius of curvature as the grating to be duplicated to be used as a test plate for the correction of the replica, and then a concave glass mirror on which the replicas are mounted and treated as just described.

At the present time the author has a number of plane platinum, nickel, and gold gratings made by the above process, which perform admirably, as well as a number of glass ones made by the hydrofluoric acid process.

I wish to express my thanks to Mr. Sparrow, who has given me much valuable aid in this work.

PHYSICAL LABORATORY
JOHNS HOPKINS UNIVERSITY
January 18, 1910

Minor Contributions and Notes

NOTE ON THE CALCIUM BANDS AT λ 6382 AND λ 6389

An important feature of sun-spot spectra is the presence of the calcium bands with heads at λ 6382 and λ 6389. Their presence was discovered by Professor Fowler. In a previous paper[1] by the writer it was shown that these groups appear in the metallic arc, burning in air under greatly reduced pressure, and their wave-lengths were measured. The purpose of the present note is to report some further observations concerning these bands.

The concave grating, inclosed arc, and the other apparatus used have all been described in former publications.[2]

In air.—No signs of these bands can be found on the plates of the spectra of the arcs in air at atmospheric pressure. The electrodes were pure metallic calcium and carbon poles filled with $Ca(OH)_2$, $(CaSO_4)_2H_2O$, $(C_3H_5O_3)_2Ca$, and $(C_7H_7SO_3)_2Ca$. They are also absent from the arcs in air at a pressure of half an atmosphere, but when the pressure is reduced to about 3 cm of mercury and less they come out strongly and the plates show that the intensities of the heads are as strong as any of the lines in this neighborhood, such as $\lambda\lambda$ 6439, 6462. This observation was made only with metallic poles. Every precaution was taken to exclude water-vapor from the inclosed arc at the low pressures by covering the bottom with a layer of phosphorus pentoxide.

In hydrogen.—Olmsted[3] found these bands present in the arc burning in hydrogen. This observation was repeated; the bands appear clearly on the plates but never with a relative intensity greater than those from the arc *in vacuo*. When steam was continually driven into the vessel the bands did not appear. Olmsted, however, found them when the steam entered through a hole in an electrode.

In nitrogen.—Nitrogen carefully prepared and thoroughly dried was admitted into the vessel until the pressure was about that of an

[1] *Astrophysical Journal*, **30**, 14, 1909.

[2] *Ibid.*, **27**, 152, 1909, and **30**, 15, 1909.

[3] *Ibid.*, **27**, 68, 1908.

atmosphere. The bands appear on the plates but never as strong as those from the arc *in vacuo*.

In sulphur dioxide the arc does not contain these radiations.

It has been suggested that these bands are due to a calcium and hydrogen compound. Although it can always be claimed that even *in vacuo* sufficient hydrogen may be liberated from the hot poles to form the "hydride" necessary for the production of these bands, nevertheless in the light of the above experiments it seems very doubtful, since they do not appear in the spectra of compounds containing hydrogen, and even in the arc in hydrogen their intensities are no greater than in the arc in air under reduced pressure. In a recent article by King[1] on the radiations from an electric furnace the same view is expressed.

Brooks[2] remarks in a very interesting paper upon the magnesium spectrum that the so-called "hydride" spectrum is still an open question. He suggests that the flutings may be due to the metal itself and not radiations from a compound, and cites the views of Hemsalech, Hartley, and Ramage in support of this idea.

So far as my observations go, they seem to indicate that these calcium bands may also be considered as true metallic radiations. The presence of hydrogen and nitrogen surrounding the arc does not influence them to any great amount; air and sulphur dioxide destroy them completely.

JAMES BARNES

BRYN MAWR COLLEGE
January 1910

FOUR STARS HAVING VARIABLE RADIAL VELOCITIES

Of the measures recently made on Bruce spectrograms the following are of immediate interest.

The place of *α Cygni* in the spectral classification has been a subject of considerable discussion, but it is properly included in Vogel's type Ia2, and is characterized by the numerous and sharp enhanced metallic lines, as was long ago pointed out by Lockyer.

[1] *Astrophysical Journal*, **29**, 381, 1909.

[2] *Proc. R. S.*, **80**, 218, 1907.

The lines are seldom blended and the spectrum is thus very satisfactory for measurement.

α Cygni ($\alpha = 20^h\ 38^m$; $\delta = +44°\ 55'$; Mag. = 1.3)

Plate	Date		G. M. T.	Taken by	No. Lines	Velocity	Quality
						km	
B 121	1901	Mar. 31	$20^h\ 52^m$	E	16	−9.1	g.
B 131		May 10	21 45	F, E	15	−6.9	g.
A 167		Aug. 14	15 30	F	12	−4.3	g.
A 168		Aug. 14	15 54	F	13	−4.4	g.
B 326	1902	April 16	20 30	A	13	−2.6	g.
B 463		Nov. 27	15 47	F, A	13	−3.0	g.
B 603	1905	July 1	17 6	F	10	−5.2	g.
A 525		Oct. 23	14 11	F	17	−0.1	v. g.
IB 1074	1907	June 1	21 39	Fox	12	−1.9	v. g.
1075		June 1	21 53	Fox	18	−2.5	v. g.
2177	1909	Nov. 8	15 18	L	16	−9.4	v. g.
2182		Nov. 19	11 28	B, L	13	−9.3	g.

A = Adams, B = Barrett, E = Ellerman, F = Frost, L = Lee. g = good, w = weak, v = very. Mr. Sullivan assisted as usual in guiding on all the plates in this note.

The only published velocity determinations of *α Cygni* are those by Vogel and Scheiner[1] from plates taken at Potsdam in 1888–1889. Using four plates which give values ranging from −4.5 to −10.4 km, Vogel derived a mean of −6.0 km, Scheiner a mean of −10.0 km. Imperfections in the apparatus and method of measurement in that early day of spectrography would, no doubt, account for this range even if the velocity of the star had been constant.

α Cygni has been casually observed with the Bruce spectrograph since 1901. All spectrograms but the last four were taken with three prisms. Camera A is a Zeiss anastigmat of 449 mm focus; B, a Hastings triple of 608 mm focus. The last four plates have the dispersion of one prism.

Plate No. B 326 was measured by Mr. Walter S. Adams, B 463 by Professor Frost. The other measures are mine. On our one-prism plates the calcium lines H and K are sharp, and they were used in the measurement, being in satisfactory agreement with the other lines. The excellent character of the lines had suggested to Mr. Frost the use of this star as a control for low-dispersion plates. On measuring the one-prism plates taken with this in view, the variable velocity became apparent, and this was confirmed by the early

[1] *Publikationen der Astroph. Obs. zu Potsdam*, **7**, Theil 1, 1892.

three-prism plates that had not been measured. The range of 9 km happens to be the same on the one-prism and three-prism plates.

The orbit of *α Cygni* should be determined from spectrograms taken with the highest possible dispersion, but this does not preclude the desirability of a careful study of the velocity derived from the H and K lines.

The velocities given above do not suggest the period.

58 Tauri ($\alpha = 4^h\ 15^m$; $\delta = +14°\ 51'$; Mag. $= 5.3$)

Plate	Date	G. M. T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 1867......	1908 Nov. 16	$20^h\ 33^m$	B	9	+41	g.
2179......	1909 Nov. 8	18 51	B	4	+32	w.
2195......	Nov. 25	17 50	F, L	11	+17	v. g.
2262......	1910 Jan. 18	14 28	L	9	+15	g.

This star, and the one following, are members of the *Taurus* stream discovered by Professor Boss;[1] this is No. 1007, the next is No. 1092, of his catalogue. From the radial velocities of three stars, γ, δ, and ϵ, as determined by Küstner, Boss has predicted velocities for the other stars of this group. His prediction of +39.9 km for *58 Tauri* seemed to be substantiated by our first plate. The last two plates, however, have proved the binary character of the star. The spectra of both *58* and 7°681 *Tauri* are classified as A in the *H. R.* notation. Each has a great many lines that apparently might be measured, but, as a matter of fact, so many of them are difficult blends that only the comparatively few lines used seemed reliable for velocity determinations. The region of the spectrum used extends from λ 4045 to $H\beta$.

It is a singular fact that of the eight spectroscopic binaries in this group that have so far been investigated here only two have thus far shown variations in velocity extending appreciably above the predicted values. These are θ^2 and *69* (upsilon) *Tauri*, found by Professor Frost.[2]

The remarkable abundance of spectroscopic binaries among the stars of this stream, pointed out (*loc. cit.*) by Mr. Frost last year, is

[1] *Astronomical Journal*, **26**, 31, 1908.

[2] *Astrophysical Journal*, **29**, 237, 1909.

confirmed by the subsequent observations. Two-thirds of those stars of which three or more spectrograms have thus far been obtained here are found to vary in radial velocity.

B. D. 7°681 *Tauri* ($\alpha = 4^h\ 34^m$; $\delta = +7°\ 40'$; Mag. $= 5.6$)

Plate	Date		G. M. T.	Taken by	No. Lines	Velocity	Quality
						km	
IB 1830......	1908	Nov. 6	$18^h\ 45^m$	B, L	10	+34	g.
1974......	1909	Feb. 1	14 30	F, L	14	+42	g.
2226......		Dec. 29	14 2	L	10	+29	g.
					7	+28	
2267......	1910	Jan. 21	14 37	B	11	+17	g

The first two values are the means of accordant duplicate measures. The second measure on plate No. 2226 is by Professor Frost. Boss predicted a radial velocity of $+41.8$ km for this star. Measures of the first plate indicated a departure from constant velocity, which succeeding plates have confirmed.

θ Pegasi ($\alpha = 22^h\ 5^m$; $\delta = +5°\ 42'$; Mag. $= 4.1$)

Plate	Date		G. M. T.	Taken by	No. Lines	Velocity	Quality
						km	
IB 803......	1906	July 13	$20^h\ 8^m$	F	3	−11	g.
810......		July 20	21 0	F	4	+ 4	v. g.
1771......	1908	Oct. 5	14 5	F, L	4	+ 8	g.
1816......		Nov. 2	13 47	L	6	−31	v. g.
1826......		Nov. 6	14 58	B	6	+10	v. g.
1841......		Nov. 9	13 8	L	6	+19	v. g.
2117......	1909	Aug. 27	17 3	B	4	−32	g.

This star has a spectrum of type Ia2 in the notation of Vogel. There are only a very few metallic lines that are strong enough to measure. Measures on the hydrogen lines were given most weight.

The binary character of this star was long ago suspected by Professor Frost upon examining the early plates. Plate No. 1816 shows a very faint component at $+62$ km.

OLIVER J. LEE

YERKES OBSERVATORY
February 16, 1910

REVIEWS

Annals of the Astrophysical Observatory of the Smithsonian Institution. Vol. II. By C. G. ABBOT AND F. E. FOWLE.

In the bolometric study of the infra-red solar spectrum conducted by the Astrophysical Observatory of the Smithsonian Institution it was found that this region of the spectrum was the seat of great terrestrial atmospheric absorption, that the intensity of absorption of the lines was variable, and that the relative intensity of energy in different parts of the spectrum changes appreciably. A thorough and detailed account of the investigations is given in the first splendid volume of the *Annals*. With the publication of these results, the work of mapping the spectrum was discontinued and attention was directed specifically toward the determination of the total solar radiation, the distribution of energy in the solar spectrum, and its variations. The present volume records in detailed form the methods and results of these and allied investigations.

Three distinct lines of observations are discussed, and they all point harmoniously and more or less clearly to the possibility of variation of the so-called solar constant. Such a variation would be of the greatest interest to astronomers, climatologists, and geologists.

Part I deals directly with the determination of the solar constant of radiation. High and low solar observations were made simultaneously with spectrobolometer and pyrheliometer at Washington and at Mt. Wilson, California. The pyrheliometer measures the total energy of the solar radiation as it reaches the instrument. The spectrobolometric observations enable the observer to estimate the losses the beam has suffered in the earth's atmosphere. The accuracy with which this estimate can be made finds noteworthy illustration in the comparison of the results from the two stations. Although the pyrheliometer at Washington showed only three-quarters of the quantity of energy recorded at Mt. Wilson, the estimation of atmospheric transmission was of such accuracy that its application brought the results into sensible accord. Early chapters give a full description of methods and apparatus, sample observations and reductions, and a discussion of the sources of error. From the last the authors conclude that separate determinations at Mt. Wilson will seldom differ by as much as 1.5 per cent. or at Washington by 3 per cent. The lower curves of Plate XV are of particular interest in showing that the

methods are of such sensitiveness that they reveal the slow variation of the solar radiation depending on the sun's varying distance. It seems unavoidable to ascribe the outstanding variations which amount to 10 to 15 per cent. to a variation in the solar constant.

In Part II there is a discussion of the dependence of terrestrial temperatures on solar radiation. From the Stefan-Boltzmann law we find that a fractional change of absolute temperature of a perfect radiator is one-fourth the fractional change of radiation which accompanies it. For the earth, if it were a perfect radiator, a change of 1 per cent. in the solar constant would produce a change in temperature of $\frac{2}{3}$°. But the curve for response in temperature-change of a perfect radiator following fluctuating insolation is more or less modified and straightened by the curves of actual terrestrial stations, much as the erratic track of the fore wheel of a bicycle is reproduced in modified form by the rear wheel. As we increase the distance between the wheels the rear wheel modifies more and more the deviations of the front wheel; so also as we go from inland meteorological stations to coast and island stations and the insulation against temperature-change exerted by great water masses is more and more potent, the response to varying insolation is more and more sluggish. It is estimated that a fluctuation of 5 per cent. in solar radiation with a period of a year would produce a change of 1° for inland and $0\overset{\circ}{.}3$ for island stations. The authors selected 47 well-distributed inland stations and examined their temperature records. Their conclusion is that there are well-marked deviations from normal mean temperatures of these stations which embrace them all. No comparison was made between the curves of temperature deviation and the solar-constant curves. Such a comparison shows, however, that the curves follow fairly well during 1903, but for no other time is there striking agreement.

Part III gives results of study of the radiation from different parts of the solar disk. Spectrobolometric observations giving the transmission coefficients for different wave-lengths at varying distances from the center of the disk show that the transparency of the solar envelope varies. Here is independent evidence of the variation of the solar constant and an indication of its cause as well. A comparison of departures from mean coefficients of transmission with solar constant results gives, however, only questionable agreement. This lack of agreement suggests the necessity of seeking another cause for the variation of radiation.

Professor C. L. Poor[1] has published three papers on "The Figure of

[1] *Astrophysical Journal*, **22**, 103, 305, 1905; Contrib. Columbia University Obs., No. 26.

the Sun. His observations indicate a variation in the diameter of about $0''.1$. Moulton[1] has shown that this would produce a prohibitive change of solar temperature in the order of 1400° C. A small oscillation might, however, exist, and he found that an oscillation of $0''.01$ would produce the variation of 10 per cent. in the solar constant first reported by Langley[2] and confirmed in this volume. Such an oscillation may well be the true cause, and if so, the investigators in any given determination catch the radiation in one point of its rapid change. For Moulton finds that the period of oscillation is short, and Emden in his *Gaskugeln* (p. 453) states: "Betrachtet man aber die Sonne in jedem Moment als adiabatische Gaskugel, so bemisst sich die Dauer ihrer Gravitationsschwingungen nur nach Stunden." If the solar radiation follows this rapid oscillation it is hopeless to expect agreement between the results of the three investigations included in the *Annals*. Between the solar atmospheric transmission coefficients and terrestrial temperatures there may be some connection. Data given here can be compared only in certain months of 1905. A longer interval is needed.

The by-products of these investigations are scarcely less important than the main results, but they can only be mentioned. The development of a standard pyrheliometer; a determination of the reflecting power of clouds, 65 per cent.; the albedo of the earth, 37 per cent.; new determinations of the solar temperature; a very interesting solar theory, constitute other material treated.

The volume contains the results of observations of the most difficult character made with extreme care. The volume is a model typographically, the plates are excellent, the arrangement and indices perfect.

PHILIP FOX

The Inequalities in the Motion of the Moon Due to the Direct Action of the Planets. (An Essay Which Obtained the Adams Prize at the University of Cambridge for the year 1907.) By ERNEST W. BROWN. Cambridge: The Cambridge Press, 1908. Pp. xii+92.

Professor Brown has spent many years on the Lunar Theory, the greater part of which is concerned with the orbit of the moon as disturbed by the sun. He has carried out the extremely laborious piece of work of calculating the perturbations produced by the sun, following the general lines marked out by Hill in his celebrated *Researches*, and his results have a degree of accuracy, when considered as a whole, not before attained.

[1] *Astrophysical Journal*, **29**, 278, 1909.

[2] *Ibid.*, **19**, 305, 1904.

It is sufficient to state that, assuming that the series converge, the computations give the position of the moon with at least as great accuracy as it can be observed. Hence, in order to test the sufficiency of the theory and incidentally to make the most rigorous demands on the law of gravitation which are capable of being made, it became necessary to add the perturbations due to other causes, such as the direct action of the planets. Brown's prize memoir is devoted to this problem.

In computing the effects of the direct action of the planets on the motion of the moon there are certain practical difficulties, the chief of which are finding the derivatives of the moon's co-ordinates with respect to its mean motion, whose numerical value is used in Brown's theory, and the fact that the sensible terms appear only at rare intervals, and are likely to be overlooked. Brown overcame the first difficulty by an ingenious method invented in 1903, and the second by the construction of a "sieve" by means of which those terms which it is necessary to compute could be separated out from the others. Words of praise for the excellence of the work are superflous when it is noted that the memoir obtained the Adams prize.

F. R. M.

Magneto- und Electrooptik, von WOLDEMAR VOIGHT. Leipzig: B. G. Teubner, 1908. 8vo, pp. xiv+396, with 75 figs. M. 14, bound.

It will be commonly conceded that there is no one living who is better qualified to present the existing status of the electron theory of the Zeeman, Faraday, and Kerr effects than is the pre-eminent Göttingen physicist whose name appears upon the title-page of this book, and who has himself made such important contributions to the theory of these phenomena.

The book was published about a year ago, and at once took its place as the most authoritative and the most complete treatment of this subject which has yet appeared. And although the field is one in which new experimental data are continually appearing and demanding a place in existing theory, there can be little doubt that for years to come Professor Voigt's book will hold its place as a most important reference work both for the experimental and the theoretical student of magneto-optics. For it contains not only all the experimental data which had appeared in this field up to the time of publication, but also an admirably clear and logical development of the whole electron theory of light, particularly in its relations to the phenomena of dispersion and absorption. One of the most noteworthy features of the book is its beautiful interweaving of theory

and experiment, and the careful working-out of numerical examples to support theoretical conclusions.

The author has avoided the notation of the vector analysis, first, because, as he says, his book is designed for the experimental as well as for the theoretical physicist, and, second, because he finds that in the problems with which he has to deal this notation offers no appreciable advantage.

Roughly speaking, the first half of the book has to do with the presentation of experimental data and the simpler aspects of the electron theory as applied to dispersion, absorption, the Zeeman and the Faraday effects; while the last half presents the theory, largely due to the author, of the more complex types of Zeeman effect, the Kerr effect, and the magneto-optics of non-isotropic media. In the last fifty pages are found the theory of the vibrations of bound electrons under the influence of an electric field.

R. A. MILLIKAN

THE UNIVERSITY OF CHICAGO

A General Index to Sidereal Messenger (Vols. 1–10), *Astronomy and Astrophysics* (Vols. 11–13), *Popular Astronomy* (Vols. 1–16). By W. W. PAYNE. Northfield, Minn., 1909. Price $1.50; bound $2.50.

This index to the journals successively edited by Professor Payne and his associates will be welcomed as an important addition to astronomical bibliography. It is arranged first by authors, and then by subjects. Natural abbreviations of one letter readily distinguish the journal in which a paper appeared. The type and presswork are excellent. A few typographical errors have been noticed, but the references to volumes and pages have doubtless been carefully checked for their accuracy.

Annuaire astronomique de l'observatoire royal de Belgique. Bruxelles: Hayez, 1909. Pp. 534, with diagrams and plates.

This excellent little book appears in the same style as in recent years. Besides its accurate data of an astronomical kind, it contains an admirable sketch by Professor Stroobant on the progress of astronomy in 1908.